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# Introduction to Bond Math

*Presentation to CDIAC*

# Agenda

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- I. What is a Bond?
- II. Key Concepts of Municipal Bonds
- III. Yield Curve
- IV. Fixed vs. Variable Rate Debt
- V. Amortization Structures
- VI. Key Calculations from a Bond Sale
- VII. Question and Answer

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**What is a Bond?**

# What is a Bond?

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- ◆ A **bond** is a debt instrument that allows issuers to finance capital needs. It obligates the issuer to pay to the bondholder the **principal** plus **interest**.
  - A buyer of the bond is the lender or investor.
  - A seller of the bond is the borrower or issuer.
- ◆ When an **investor** purchases a bond, he is lending money to a government, municipality, corporation, federal agency or other entity.
- ◆ In return for buying the bond, the issuer promises to pay the investor a specified rate of interest during the life of the bond and to repay the face value of the bond (the principal) when it “matures,” or comes due.
- ◆ In addition to operating covenants, the loan documents require issuer to spend the bond proceeds for the specific projects.
- ◆ Among the types of bonds an investor can choose from are: U.S. government securities, municipal bonds, corporate bonds, mortgage and asset-backed securities, federal agency securities and foreign government bonds, among others.
- ◆ A bond can also be thought of as a contract between the issuer and investor. This contract specifies, for example, the terms of the bonds, the funds from which debt service will be paid and any operating covenants.

# Source of Repayment for Debt Service

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- ◆ **General Obligation (“GO”) Bonds** are secured by a pledge of the issuer’s full faith, credit and taxing power. The “full faith and credit” backing of a General Obligation bond implies that all sources of revenue, unless specifically excluded, will be available to pay debt service on the bonds.
  
- ◆ **Appropriation Bonds** are secured by a “promise to pay” with legislatively approved appropriations. These are generally supported by the General Fund of issuer, unlike General Obligation bonds where funds are often not paid from the General Fund.
  - Examples include Certificate of Participation (COPs) and Leased Revenue Bonds (LRBs).
  
- ◆ **Revenue Bonds** are payable from a specific stream of revenues, such as a user fee or dedicated tax, and are not backed by the full faith and credit of the issuer. They are issued to finance specific enterprises or projects and are usually secured solely by revenues from those projects. Revenue bonds can generally be grouped into the following categories:
  - Utilities
  - Higher Education, Healthcare and Other Not-For-Profit
  - Housing
  - Transportation
  - Industrial Development, Pollution Control, and Other Exempt Facility Bonds
  - Securitized Revenue Bonds

# Bond Covenants and Other Security Features of Revenue Bonds

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- ◆ **Rate Covenants** - Under a rate covenant, the issuer pledges that rates will be set at a level sufficient to meet operation and maintenance expenses, renewal and replacement expenses, and debt service. An alternative form of rate covenant requires that rates be set so as to provide a safety margin above debt service, after operation and maintenance expenses are met.
  - Example: *“The Board will fix, charge and collect fees so that the Revenues will at all times be sufficient in each Fiscal Year to pay Operating and Maintenance Expenses and to provide funds at least equal to 115% of (1.15 times) the Principal and Interest Requirements....”*
  
- ◆ **Additional Bonds Test (ABT)** - Protects existing bondholders from the risk that their security will be diluted by the issuance of additional debt. The Additional Bonds Test must be met by the issuer in order to borrow additional debt secured by the same revenue source as the outstanding bonds.
  - Example: *“The Net Revenues in each of the two Fiscal Years immediately preceding the date of issuance of such proposed Additional Bonds must be equal to at least 130% of the estimated Annual Debt Service for the year following the proposed issuance.”*

# Bond Covenants and Other Security Features of Revenue Bonds (cont.)

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- ◆ **Debt Service Reserve Fund** - Provides a cushion to make timely debt service payments in the event of temporary adversity. Federal law limits the amount of tax-exempt bond proceeds that can be used to fund the debt service reserve fund to the lesser of:
  - *10% of the principal amount of the issue;*
  - *Maximum annual debt service; and*
  - *125% of average annual debt service on an issue.*
  - May also be required for appropriation debt.
  - Many times a DSRF is not required for highly rated credits (e.g. UC Regents and CSU).
  
- ◆ **Other Covenants** - Additional covenants might include a provision for insuring the project, a review by an independent auditor, or a prohibition against the sale of the project's facilities prior to repayment of outstanding debt, among others.

# Uses of Bond Proceeds

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## **New Money**

Bonds issued to provide new or additional funding for a project.

## **Refunding**

Bonds issued to refinance certain existing bonds (proceeds used to repay old bonds). Refundings can be used to produce savings, restructure debt service or release the issuer from restrictive operating covenants.



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## **Key Concepts of Municipal Bonds**

# Key Concepts – Basic Terminology

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- ◆ Principal
- ◆ Maturity
- ◆ Serial Bonds
- ◆ Term Bonds and Sinking Funds
- ◆ Coupon
- ◆ Yield
- ◆ Price
- ◆ Interest
- ◆ Debt Service
- ◆ Original Issue Discount
- ◆ Original Issue Premium
- ◆ Bond Proceeds
- ◆ Capital Appreciation Bonds
- ◆ Callable Bonds
- ◆ Insurance
- ◆ Bond Conventions

# Principal and Maturity

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- ◆ **Maturity** - Date on which principal payments are due
  - Typically, maturity dates are generally no longer than 30 years
  - Most bond issues have principal maturing each year until the final maturity date of the series
- ◆ **Principal** - Also known par amount, or face value, of a bond to be paid back on the maturity date
  - Typically, bonds are sold in \$5,000 principal denominations, often \$100,000 for variable rate bonds

<b><u>Maturity Date</u></b>	<b><u>Principal</u></b>
1/1/2011	\$8,705,000
1/1/2012	\$9,005,000
1/1/2013	\$9,325,000
1/1/2014	\$9,685,000
1/1/2015	\$10,170,000
1/1/2016	\$10,705,000
<b>Total</b>	<b>\$57,595,000</b>

# Serial and Term Bonds

- ◆ Bonds can either mature annually (serial bonds) or as term bonds.
- ◆ A term bond is a series of sequential amortizations. Payments of principal prior to the term bond's final maturity are referred to as sinking fund payments.

<b>Maturity Date</b>	<b>Principal</b>	<b>Coupon</b>	
1/1/2011	\$8,705,000	3.50%	} <b>Serial Maturities</b>
1/1/2012	\$9,005,000	3.50%	
1/1/2013	\$9,325,000	3.90%	
1/1/2014	\$9,685,000	5.00%	
1/1/2015	\$10,170,000*	5.25%	} <b>Term Bond</b>
1/1/2016	\$10,705,000*	5.25%	
<b>Total</b>	<b>\$57,595,000</b>		

\*Sinking fund payment

# Coupon, Interest and Debt Service

- ◆ **Coupon** - Percentage rate (based on principal/par amount) of annual interest paid on outstanding bonds
  - Can be fixed or variable
- ◆ **Interest** - Cost of borrowing money for the issuer
  - Usually paid periodically
    - Semi-annually for fixed-rate bond
    - More frequently for variable-rate bonds
  - Interest is calculated by multiplying principal by coupon (adjusted for length of period between interest payments)
- ◆ **Debt Service** - Sum of all principal and interest on a bond

<b>Year</b>	<b>Principal</b>	<b>Coupon</b>	<b>Interest</b>	<b>Debt Service</b>
2010			\$2,563,713	\$2,563,713
2011	\$8,705,000	3.50%	\$2,563,713	\$11,268,713
2012	\$9,005,000	3.50%	\$2,259,038	\$11,264,038
2013	\$9,325,000	3.90%	\$1,943,863	\$11,268,863
2014	\$9,685,000	5.00%	\$1,580,188	\$11,265,188
2015	\$10,170,000	5.25%	\$1,095,938	\$11,265,938
2016	\$10,705,000	5.25%	\$562,013	\$11,267,013
<b>Total</b>	<b>\$57,595,000</b>		<b>\$12,568,466</b>	<b>\$70,163,466</b>

# Bond Pricing

- ◆ **Price** – Discounted present value of debt service on an individual maturity. Debt service is calculated using the coupon and discounted at the yield.

<b>Date</b>	<b>Principal</b>	<b>Coupon</b>	<b>Interest</b>	<b>Debt Service</b>	<b>Present Value to 1/1/2009 at 3.82%</b>
1/1/2009					
7/1/2009			\$1.75	\$1.75	\$1.72
1/1/2010			\$1.75	\$1.75	\$1.69
7/1/2010			\$1.75	\$1.75	\$1.65
1/1/2011	\$100.00	3.50%	\$1.75	\$101.75	\$94.33
<b>Total</b>	<b>\$100.00</b>		<b>\$7.00</b>	<b>\$107.00</b>	<b>\$99.39</b>

<b>Price</b>	<b>\$99.39</b>
<b>Par Amount</b>	<b>\$8,705,000.00</b>
<b>Purchase Price</b>	<b>\$8,651,812.45</b>

# Bond Pricing (cont.)

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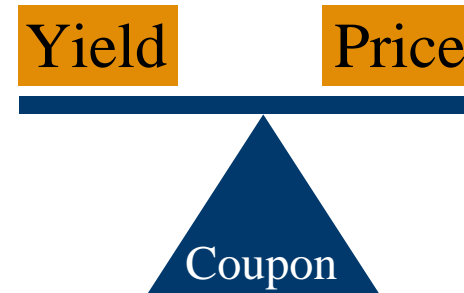
- ◆ As a result, price and yield move in opposite directions.



# Par, Discount and Premium Bonds

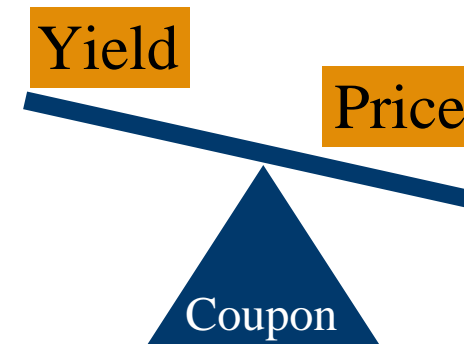
## ◆ Par Bonds

- Coupon equals yield
- Purchase price equals principal amount



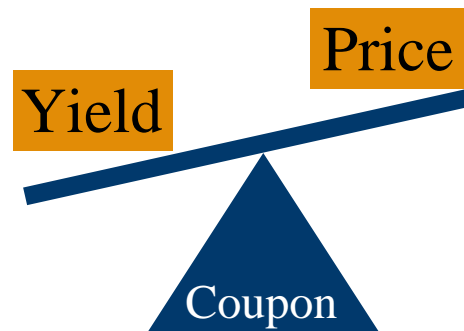
## ◆ Discount Bonds

- Coupon less than yield
- Purchase price less than principal amount



## ◆ Premium Bonds

- Coupon greater than yield
- Purchase price greater than principal amount





# Par, Discount and Premium Bonds (cont.)

<b>Maturity Date</b>	<b>Principal</b>	<b>Coupon</b>	<b>Yield</b>	<b>Price</b>	
1/1/2011	\$8,705,000	3.50%	3.82%	99.389	} Discount Bonds
1/1/2012	\$9,005,000	3.50%	3.85%	99.017	
1/1/2013	\$9,325,000	3.90%	3.90%	100.000	} Par Bond
1/1/2014	\$9,685,000	5.00%	3.94%	104.768	} Premium Bonds
1/1/2016	\$20,875,000	5.25%	4.02%	107.440	
<b>Total</b>	<b>\$57,595,000</b>				

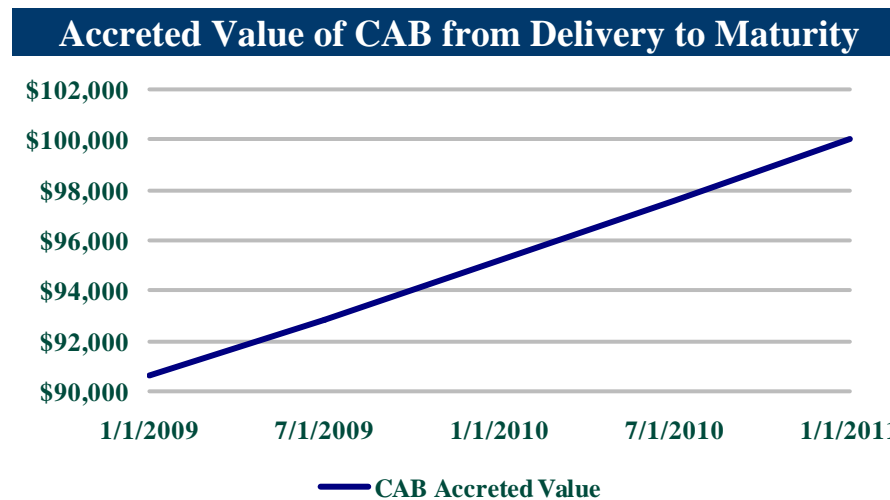
# Original Issue Discount and Original Issue Premium

Key Concepts of Municipal Bonds

<b>Maturity Date</b>	<b>Principal</b>	<b>Price</b>	<b>Original Issue Premium</b>	<b>Original Issue Discount</b>	<b>Proceeds</b>
1/1/2011	\$8,705,000	99.389		(\$53,188)	\$8,651,812
1/1/2012	\$9,005,000	99.017		(\$88,519)	\$8,916,481
1/1/2013	\$9,325,000	100.000			\$9,325,000
1/1/2014	\$9,685,000	104.768	\$461,781		\$10,146,781
1/1/2016	\$20,875,000	107.440	\$1,553,100		\$22,428,100
<b>Total</b>	<b>\$57,595,000</b>		<b>\$2,014,881</b>	<b>(\$141,707)</b>	<b>\$59,468,174</b>

# Capital Appreciation Bonds (CABs)

- ◆ CABs pay no periodic interest until maturity. The bonds accrete in value as interest accrues.
  - Usually sold as serial bonds, but can be structured as term bonds.
- ◆ At maturity an amount equal to the initial principal invested plus the interest earned, compounded semiannually at the stated yield, is paid.
- ◆ They are sold in denominations of less than \$5000 representing their present value and pay \$5000 at maturity.
- ◆ Though CABs are often more expensive (sold at a higher yield) than current interest bonds, they are used to achieve particular debt service patterns.
  - Example: A CAB maturing in 2011 may have a par amount of \$90,595 but will have a value of \$100,000 when it matures. The difference between \$100,000 and \$90,595 represents the interest on the bond.



# Callable Bonds

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- ◆ Callable bonds can be redeemed by an issuer before their actual maturity on and after a specified call date (an optional redemption provision).
  
- ◆ Many times, fixed-rate bonds will be callable 10 years after issuance at a price of par. Historically, many municipal bonds were sold with 10-year call features where the bond was callable at 102 and declined to par by the 12th year.
  
- ◆ Municipal bonds are sold with embedded call features to provide restructuring flexibility and/or refinancing savings in the future.
  
- ◆ Investors charge the issuers for this flexibility – through a higher yield and lower price – thereby increasing the cost of the financing at the time of issuance.
  - Issuers need to weigh this increased flexibility and the possibility of savings down the road against this increased cost.

# Bond Insurance

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- ◆ Issuers purchase bond insurance in order that debt service will be paid even if there are insufficient revenues.
  - In exchange for this, investors will pay a higher price (lower yield) for an insured bond.
  
- ◆ Premium paid upfront, based on original debt service schedule; no credits for refundings or early repayment of bonds.
  
- ◆ Payments by insurer are a “loan” or an “advance” that have to be paid back
  - Not like property or health insurance
  - A form of “credit enhancement”
  
- ◆ The cost of an insurance policy needs to be compared to the observed market spread between insured and uninsured bonds. It makes sense to only insure those maturities for which the cost of the policy is less than 'cost' of issuing uninsured bonds.
  
- ◆ The market for bond insurance has changed significantly over the last year. For example, several insurers have been downgraded. Also, it is unclear what the effects of Moody’s move to a Global Scale will be on bond insurance.

# Bond Conventions

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## ◆ Basis Point

- Yields on bonds are usually quoted in terms of basis points, with one basis point equal to one one-hundredth of 1 percent.
  - $.50\% = 50$  basis points

## ◆ Day Count

- 30/360
  - Usually for tax-exempt fixed rate bonds
- Actual/Actual
  - Usually for tax-exempt variable rate bonds

## ◆ Pricing

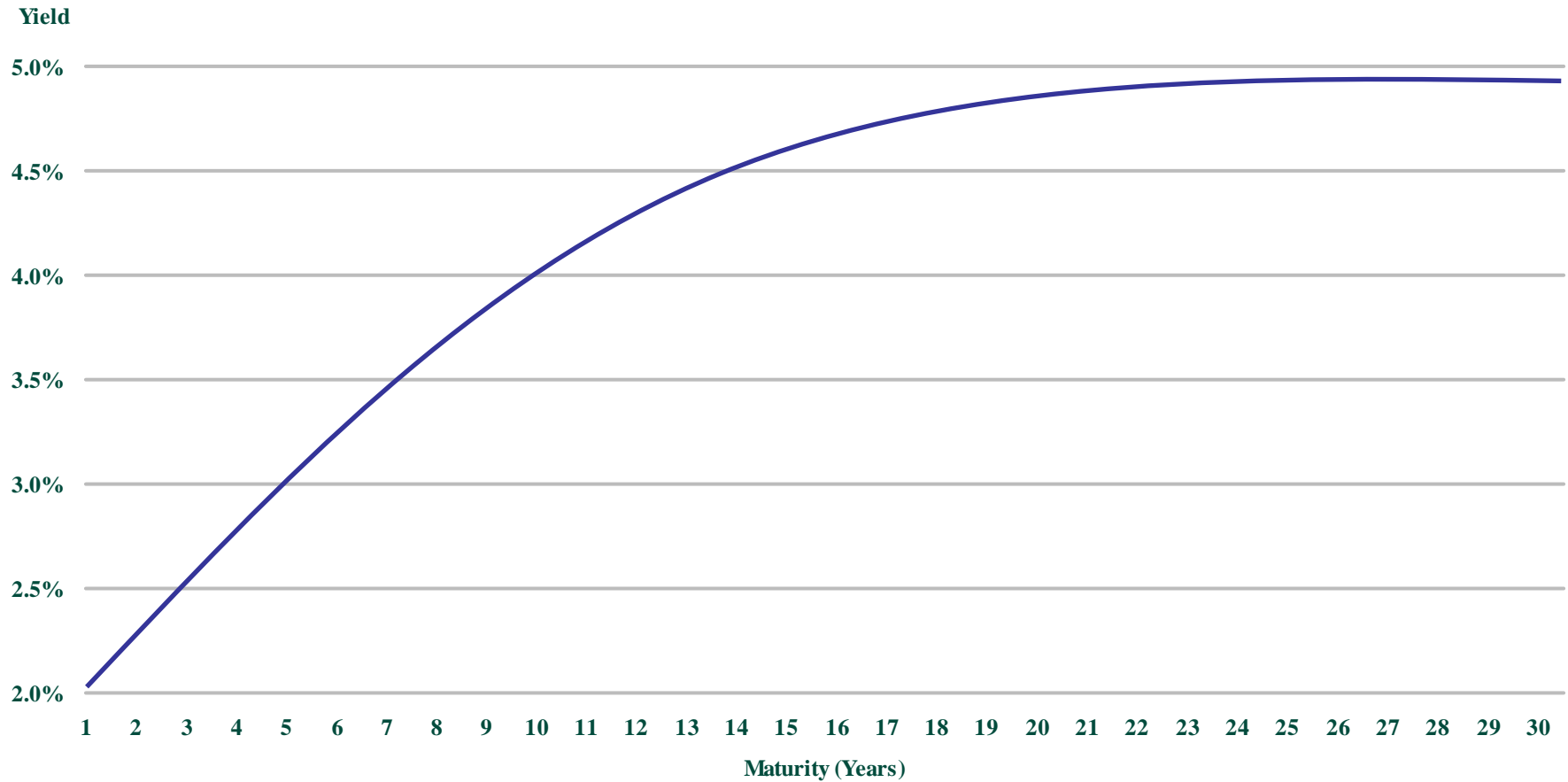
- Truncate to 3 decimals

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**Yield Curve**

# Yield Curve: Normal

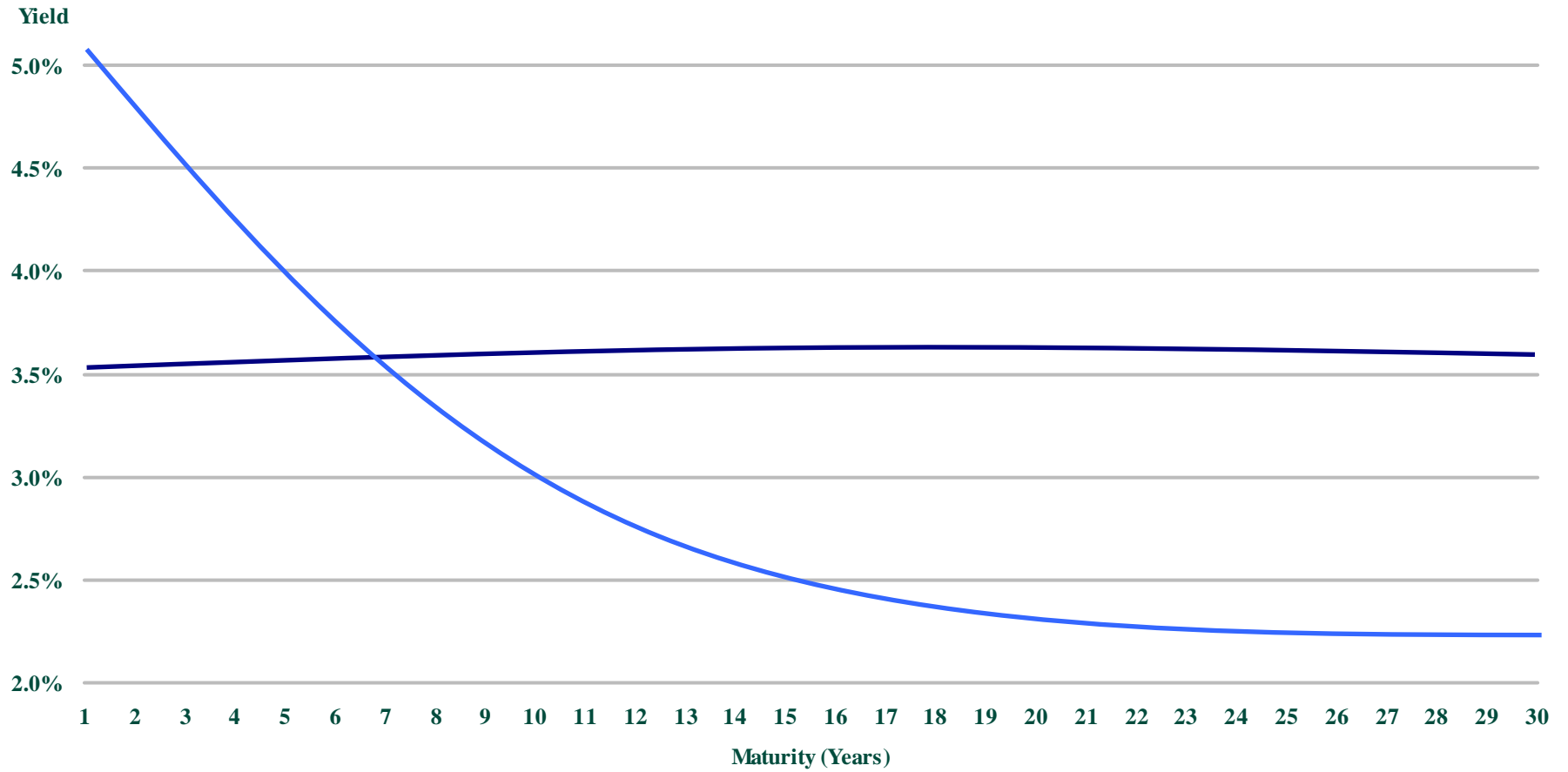
## Upward-Sloping (Normal) Yield Curve





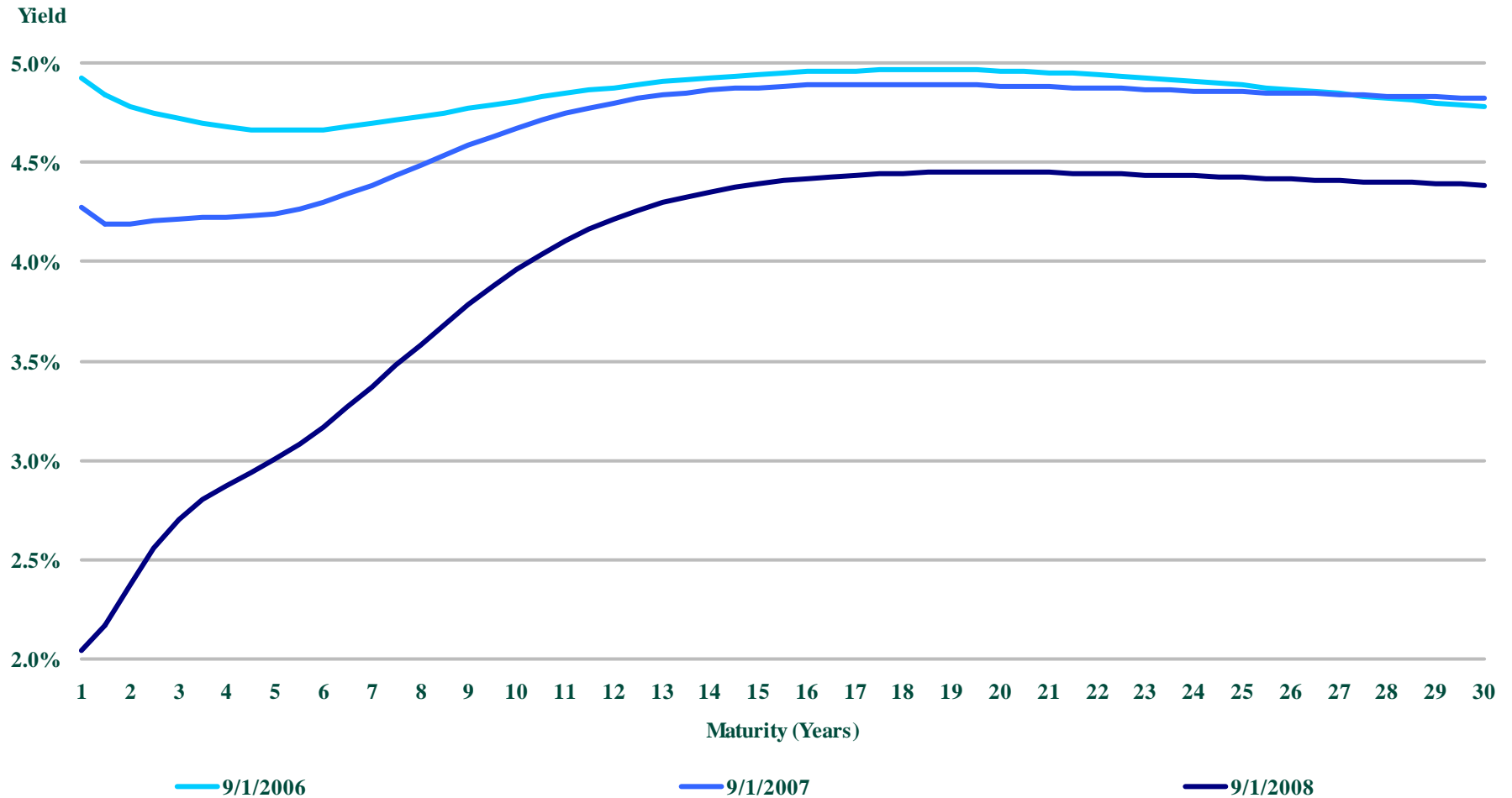
# Yield Curves: Flat and Inverted

## Flat and Inverted Yield Curves



# Current Yield Curve Compared to Yield Curves from One and Two Years Ago

U.S. Treasury Yield Curve: 9/1/2006, 9/1/2007 and 9/1/2008

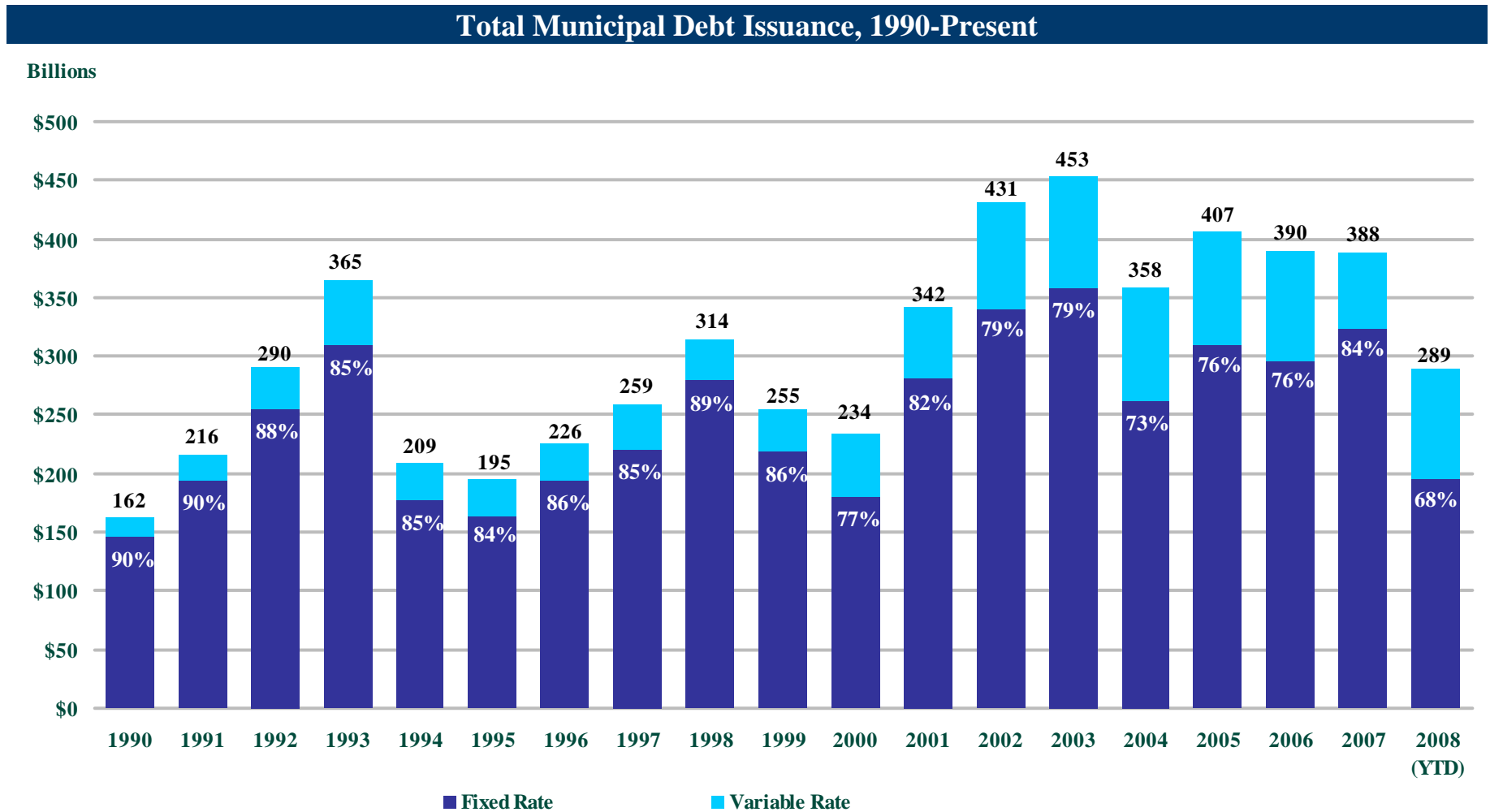


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## **Fixed vs. Variable Rate Debt**

# Fixed and Variable Rate Debt Issuance

Fixed vs. Variable Rate Debt



# Fixed vs. Floating-Rate Bonds

## Fixed Rate Bonds

### Advantages

- No Interest Rate Risk - Budget Certainty
- No Ongoing Credit Support Needed
- Traditional Investors Include: Bond Funds, Insurance Companies, Arbitrage Accounts, Trust Departments and Retail Investors

### Disadvantages

- Higher Initial and Expected Interest Expense
- Less Flexible Call Feature than Floating Rate Bonds
- Potentially Higher Issuance Costs

- ◆ Fixed rate financings remain the most common approach in the current market.

## Variable Rate Bonds

### Advantages

- Easy to Restructure
- Lower Expected Cost of Capital
- Used to Diversify Debt Portfolio
- Traditional Investors Include: Money Market Funds, Corporations and Retail Investors

### Disadvantages

- Interest Rate Risk
- Budgeting Uncertainty
- Unpredictable Pricing of Support Costs
- Additional Administrative Involvement

- ◆ Given the Fed's recent rate increases, variable rates have increased from their historical lows two years ago, with SIFMA recently resetting at 1.84%. This compares to a 20-year average of 3.15%.

# Credit Enhancement for VRDBs

- ◆ **Credit enhancement** is a means of substituting the credit of the issuer with that of a higher rated third party guarantor.
  - Similar to insurance in the case of fixed-rate bond, credit enhancement improves the marketing for bonds.
  - Credit enhancement typically takes the form of bond insurance or letters of credit (LOC).

## Bond Insurance

- ◆ Several well-established bond insurers.
- ◆ Premium is based on projected total debt service and paid up-front as a one time fee.
- ◆ In effect for life of bond issue.

## Letters of Credit (LOC)

- ◆ Typically provided by commercial banks.
- ◆ Premium is based on amount of debt outstanding and paid over time.
- ◆ Most LOCs carry an initial term shorter than the term of the bonds and must be renewed or replaced at each expiration date.

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## **Amortization Structures**

# Alternate Amortization Structures

- ◆ Issuers can use amortization structures to shape their overall debt structure pattern.

## Level Principal

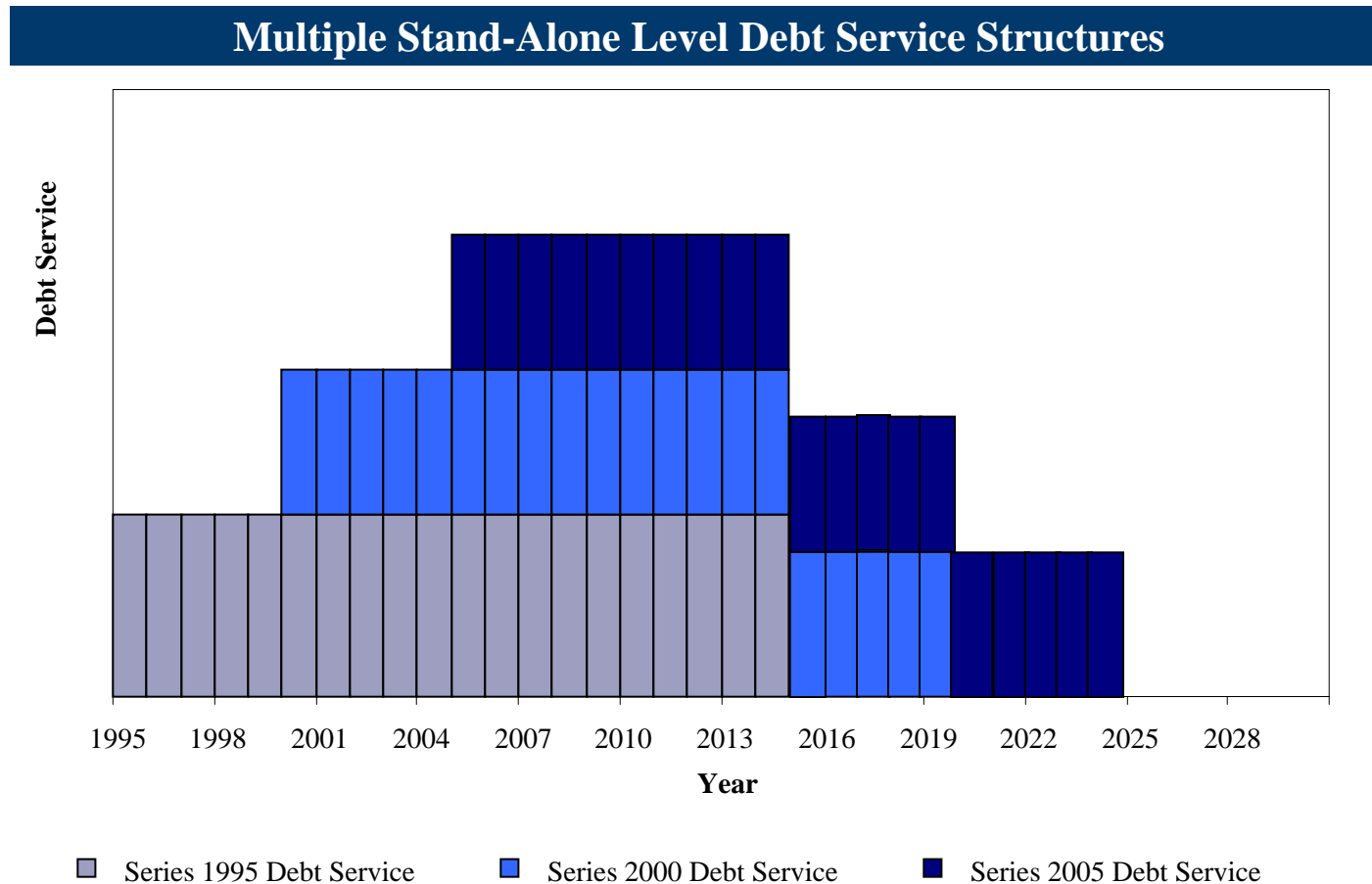
Maturity Date	Principal	Interest	Total Debt Service
1/1/2010		\$2,538,905	\$2,538,905
1/1/2011	\$9,620,000	\$2,538,905	\$12,158,905
1/1/2012	\$9,620,000	\$2,202,205	\$11,822,205
1/1/2013	\$9,620,000	\$1,865,505	\$11,485,505
1/1/2014	\$9,615,000	\$1,490,325	\$11,105,325
1/1/2015	\$9,615,000	\$1,009,575	\$10,624,575
1/1/2016	\$9,615,000	\$504,788	\$10,119,788
<b>Total</b>	<b>\$57,705,000</b>	<b>\$12,150,208</b>	<b>\$69,855,208</b>

## Level Debt Service

Maturity Date	Principal	Interest	Total Debt Service
1/1/2010		\$2,563,713	\$2,563,713
1/1/2011	\$8,705,000	\$2,563,713	\$11,268,713
1/1/2012	\$9,005,000	\$2,259,038	\$11,264,038
1/1/2013	\$9,325,000	\$1,943,863	\$11,268,863
1/1/2014	\$9,685,000	\$1,580,188	\$11,265,188
1/1/2015	\$10,170,000	\$1,095,938	\$11,265,938
1/1/2016	\$10,705,000	\$562,013	\$11,267,013
<b>Total</b>	<b>\$57,595,000</b>	<b>\$12,568,463</b>	<b>\$70,163,463</b>

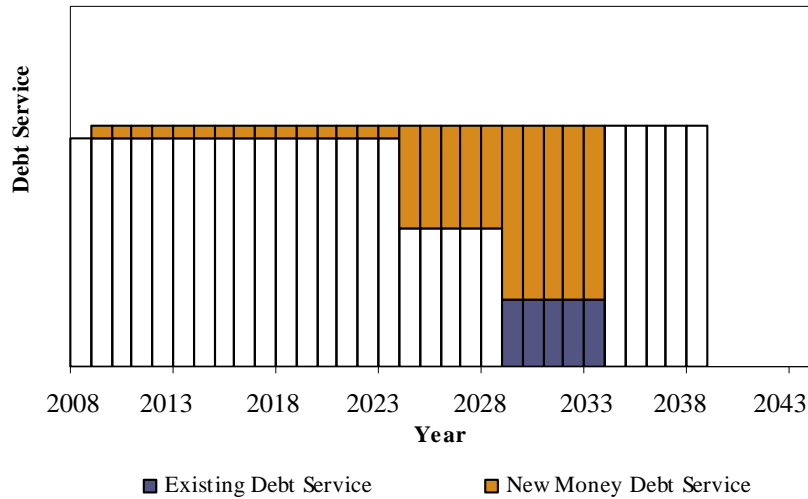


# Impact of Issuing Multiple Stand-Alone Level Debt Service Issues Over Time

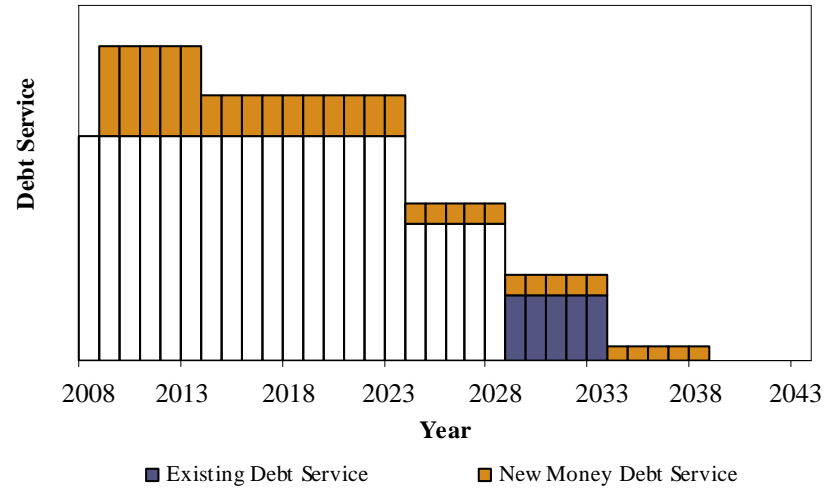


# Principal Amortization Options

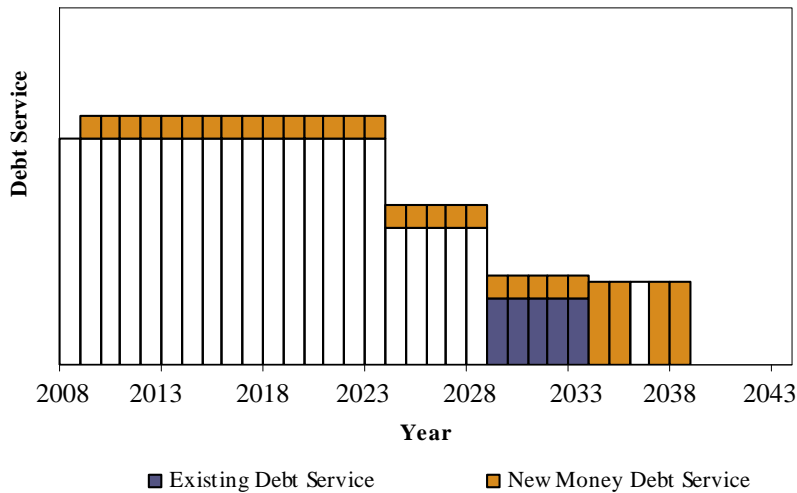
## Wrapped Debt Service Structure



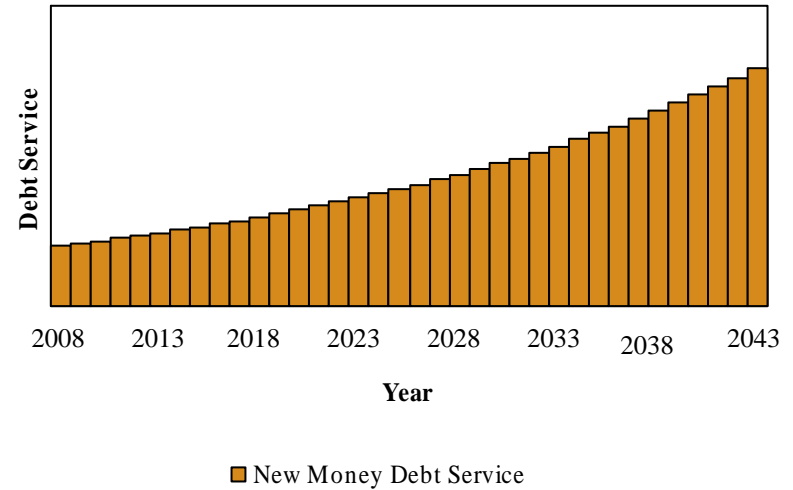
## Accelerated/Front-Loaded Debt Service Structure



## Deferred/Back-Loaded Debt Service Structure



## Increasing Debt Service Structure



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## **Key Calculations from a Bond Sale**

# Key Calculations From a Bond Sale

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- ◆ Sources and Uses of Funds
- ◆ Issuance Expenses
- ◆ Net Debt Service Schedule
- ◆ Yield Calculations

# Sources and Uses of Funds

Key Calculations from a Bond Sale

## Sources:

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Bond Proceeds	
Par Amount	\$57,595,000
Net Premium	1,873,174
<b>Total Sources</b>	<b>\$59,468,174</b>

## Uses:

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Project Fund Deposit	\$50,000,000
Other Fund Deposits	
Debt Service Reserve Fund	5,946,817
Capitalized Interest Account	2,489,242
Delivery Date Expenses	
Costs of Issuance	500,000
Underwriter's Discount	387,975
Bond Insurance	140,327
Other Uses of Funds	
Additional Proceeds	3,813
<b>Total Uses</b>	<b>\$59,468,174</b>

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# Issuance Expenses

Key Calculations from a Bond Sale

## Borrower's Costs of Issuance

Rating Agency Fees

Issuer/ Authority Fee

Bond Counsel Fee

Borrower's Counsel Fee

Trustee Fees

Auditor's Fee

Printing and Mailing Costs

Miscellaneous and Contingency

## Components of Underwriters' Discount

Takedown

Management Fee

Underwriters' Counsel

DTC

CUSIP

BMA Assessment

Dalcomp

Electronic Order Entry

Dalcomp Wire Charge

Cal PSA

CDIAC

Day Loan

Out-of-Pocket and Closing Costs

Verification Agent (if refunding)

# Net Debt Service Schedule

Key Calculations from a Bond Sale

<b>Maturity Date</b>	<b>Principal</b>	<b>Coupon</b>	<b>Interest</b>	<b>Gross Debt Service</b>	<b>Capitalized Interest</b>	<b>Net Debt Service</b>
1/1/2010			\$2,563,713	\$2,563,713	\$2,563,713	
1/1/2011	\$8,705,000	3.50%	\$2,563,713	\$11,268,713		\$11,268,713
1/1/2012	\$9,005,000	3.50%	\$2,259,038	\$11,264,038		\$11,264,038
1/1/2013	\$9,325,000	3.90%	\$1,943,863	\$11,268,863		\$11,268,863
1/1/2014	\$9,685,000	5.00%	\$1,580,188	\$11,265,188		\$11,265,188
1/1/2015	\$10,170,000	5.25%	\$1,095,938	\$11,265,938		\$11,265,938
1/1/2016	\$10,705,000	5.25%	\$562,013	\$11,267,013		\$11,267,013
<b>Total</b>	<b>\$57,595,000</b>		<b>\$12,568,463</b>	<b>\$70,163,463</b>	<b>\$2,563,713</b>	<b>\$67,599,750</b>

# Yield Calculations

- ◆ **Yield** is the discount rate at which the present value of future debt service payments are equal to the proceeds of the issue.
- ◆ The most common measures of the borrowing cost of a bond issue are the arbitrage yield, true interest cost (TIC) and all-in TIC.
- ◆ For short or non-callable issues, each is differentiated by which costs it takes account of. For example...

	Arbitrage Yield	TIC	All-In TIC
Par Value	\$57,595,000	\$57,595,000	\$57,595,000
+ Premium (Discount)	1,873,174	1,873,174	1,873,174
- Credit Enhancement/Insurance	-140,327	-140,327	-140,327
- Underwriter's Discount		-387,975	-387,975
- Cost of Issuance Expense			-500,000
Net Proceeds	\$59,327,847	\$58,939,872	\$58,439,872



# Yield Calculations for a Bond Issue

◆ In this example, the debt service used to calculate the Arbitrage Yield, TIC and All-In TIC are the same. The difference between them is the 'target' value.

	Arbitrage Yield	TIC	All-In TIC
Discount Rate*	3.98%	4.14%	4.34%
1/1/2008	-\$59,327,847	-\$58,939,872	-\$58,439,872
7/1/2008	1,281,856	1,281,856	1,281,856
1/1/2009	1,281,856	1,281,856	1,281,856
7/1/2009	1,281,856	1,281,856	1,281,856
1/1/2010	9,986,856	9,986,856	9,986,856
7/1/2010	1,129,519	1,129,519	1,129,519
1/1/2011	10,134,519	10,134,519	10,134,519
7/1/2011	971,931	971,931	971,931
1/1/2012	10,296,931	10,296,931	10,296,931
7/1/2012	790,094	790,094	790,094
1/1/2013	10,475,094	10,475,094	10,475,094
7/1/2013	547,969	547,969	547,969
1/1/2014	10,717,969	10,717,969	10,717,969
7/1/2014	281,006	281,006	281,006
1/1/2015	10,986,006	10,986,006	10,986,006

\* Also known as the Internal Rate of Return, or IRR.

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**Question and Answer**

# Questions and Answers

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