## 10/7/11 CDIAC Webinar - Bond Math II Transcript

The following transcripts attempt to capture the presentation of the Bond Math II webinar, however some areas of text may not be complete due to transcription issues; CDIAC has made an effort to correct areas of the transcript.

## Pages 1 \& 2:

Welcome to the second webinar in a two-part series on Bond Math. This is also the second webinar offered by CDI AC in this expanded format. CDIAC has offered classroom style educational workshops for debt and investments for decades. This webinar series compliments the comprehensive classroom curriculum by focusing on specific topics that may only briefly be considered in the traditional format. Today we are going to continue to examine some of the fundamental concepts underlying bond structuring and analysis. The first part of the Bond Math series addressed reading debt service schedules, understanding structuring options and reading finance reports. Today's webinar will build upon those concepts of bond math and will include yield curves, bond pricing, bond redemptions and accrued interest and other topics.

## Pages 3 \& 4:

Our presenters today are Robert Friar and Ken Fullerton. Mr. Friar is a managing director with PFM located in Lafayette Colorado. Mr. Friar joined PFM in 2010, prior to that he worked with the financial advisory firm he co-founded Fullerton and Friar. Mr. Friar has worked both as underwriter and financial advisor; he is currently responsible for assisting clients in the structuring and financing of complex airport projects. He's also worked on a variety of other projects including public power financings in Arizona employment financing in Louisiana and large airport expansions in Chicago among other things.

Mr. Fullerton joined PFM in 2010. Prior to that he worked at the financial advisory firm he co-founded, with Robert Friar, Fullerton \& Friar, Inc. Mr. Fullerton has been involved in all aspects of municipal finance since beginning his career in 1983. He currently specializes in financial advisory to airport clients. He has been involved in over 20 financings in airports in all parts of the country including Chicago, New York, Washington, Tampa, San Jose, Oakland, Salt Lake, Reno, Columbus, Providence, Tampa, I guess he does not get home much apparently.

Before we begin today's webinar we are going to go over a couple of housekeeping items. If you were participating in the first webinar you should be familiar with the screen on the top right hand side of the screen you will see the feedback button and if you have any problems with either the pace or the volume, you can use those buttons to let us know, and we will adjust the program accordingly. Likewise to the top left of the screen you will see the Q \& A function this will give you an opportunity to ask questions, during the course of the webinar and your questions will not be viewed by any of the other participants but the presenters will be able to respond to them during the course of their presentation or at the end of the presentation. The last housekeeping issue I wanted to address and it is new for those who participated last week's webinar is we have added some visual effects to our program. On the right-hand side of my screen, and if you minimize the screen you will see the speakers in video. If at any time you want to switch between the speakers, you will have to scroll down using the view current speakers and you can switch between each of the speakers to keep track the presenter speaking at the current time.

## Polling Questions not included in power point presentation:

Before we begin I want to conduct a couple of polling questions to get an idea for the speakers and for CDIAC.

First polling question: Who is participating in today's webinar.
Okay, thank you.
Second polling question: Now we are going to take a look at who is participating today and their affiliation, either as private sector or a public agency. We will tally this up quickly. We've got about 80\% reporting public affiliations, with $21 \%$ reporting private affiliations. Okay.

Third polling question: If you are public agencies who are you affiliated with, state, federal, city, county, special district, school or other.

We will take just a moment to participants to respond and we are now detailing these results. We are at about $33 \%$ state participation, with $26 \%$ between City and County participation.

Thank you for participating in our polling questions as this will help us understand how many people are actually participating in the webinar as we know that a number of people are registered but have more than one person in the office listening.

Fourth polling question: So let us know how many people are participating at your location. Okay, we've got a couple with more than five. Just a few more questions.

Fifth polling question: What is your understanding of bond economics, bond yields and pricing? Again, this will help the presenters structure their presentation. It appears that the majority of you have some understanding of our topic today. I also want to suggest that if you have any issues and we have resolved them, and if you provided feedback to us, go ahead and turn your feedback button back to green so that we know there aren't any outstanding issues on the technical side for us to address further. With that being said, I'm going to turn it over (and I probably took more time than I expected), to Robert and Ken, take it away.

Pages 5 \& 6: Good Morning this is Ken Fullerton and I would like to begin by thanking CDIAC for the invitation to participate in this program and I think it is both fun to present information like this and also to be on the cutting edge of technology. We have put together a presentation which is intended to address part two in the Bond Math series. We know that our presenters from last week from PRAG in Bond Math Part One series related to structuring an individual bond issue. However, what we want to talk about today are various topics related to the pricing and trading in bonds, and the pricing of bonds in connection with new bond sales as PRAG described last week. More specifically relevant concepts such as buying or selling bonds in the secondary market. I would like to note that the concepts that you discuss are also true when you discuss municipal bonds, but for those of you in treasury functions, and may purchase or sell other types of securities, such as US treasuries they are equally applicable, I meant to say first, it is equally applicable between municipal bonds. I will provide a general overview in the beginning of the presentation and then Robert will go into the more technical information in our presentation. I will also note particularly in parts two through five of our presentation as seen on this topic slide six there are various technical concepts that are of substance we will try not to get too weighted down and highly technical with the math piece of the presentation. However, if any of you
who have questions in that regard, and would like to forward the concepts to us please submit your questions as we go along and we will try to answer those as Robert is one of the most experienced quantitative experts in the Municipal Bond business and he has been in the business since 1983.

The concepts we are going to talk about today are concepts Robert has been addressing in his own formulas, and models that will achieve the kind of analysis we are talking about. In addition, we will describe particularly in topic part five that a lot of the analytical computations today can be done with functions that are embedded in Excel and Robert will show you how to do that, if you want to wade into the original math Robert is certainly the right guy to ask.

Pages 7 \& 8: Whether we do it in context or a presentation we will open up and talk about to you about the yield curve as I'm sure most of you are familiar with but it's a very fundamental concept that relates to bond pricing and in general, the yield curve as we've shown on slide 7 is a curve on a graph in which the yield of fixed interest securities is plotted against the length of time you have until they run to maturity. Basically at any given day in the market for a different bond, be it a certain municipal bond or treasury, bonds of the same type with different maturities are likely expected to yield different rates of return and types of yields as in most cases the longer the maturity of the bond the higher that yield will be. This is noted under normal conditions like I just said; the longer maturity bonds are going to confirm your yields.

This diagram is intended to be illustrative and is showing you that the normal shape of the yield curve which is upwardly sloping. Also talking about different market conditions the yield curve may take different shapes, but in most cases including today's case we have an upwardly sloping yield curve. Now there are a couple of reasons that economists and bond traders try and explain the reasons why the yield curve is generally upward sloping. Two of the most common reasons are the liquidity premium and the other being market expectations. The liquidity premium generally means an investor holding a longer maturity bond faces more uncertainty of what might happen in the market during the period he is holding that bond assuming the investor wants to hold it until maturity. It is also true that the holder of the longer maturity bond wants to sell it prior to maturity will have greater market risk to what the market of that bond might be on the day they want to sell it. Both of those translated to reasons why the holder of a long term bond may have less overall liquidity and therefore may demand a higher yield to be compensated for the risk factors. Secondly there is generally a market expectation that in the market over time interest rates are going to be increasing. If that is true, that means a holder of the short term bond that is maturing and needs to reinvest may find the market is at a higher rate than they could of initially if the bond was a long term bond. So the holder of the long term bond as compensation for that would demand a higher yield. This is all based on the theory that the holder of a short term bond if they reinvest sequentially to get to the same position as the original purchaser of the long term bond that their overall yield would be basically the same. Again these things do not always prove out in reality, but they are part of the market theories as to why we have an upwardly slopping yield curve. If any point along the way you have questions please let me know.

The other point that we want to cover is that there are situations where the yield curve takes different shapes than the upwardly sloping yield curve.

Pages 9 \& 10: Show an example that is unusual but can occur and is called a flat yield curve. This is basically where long term interest rates and short term interest rates and everything in between are
basically the same and if you graph that you get a nice flat line on the yield curve and is obviously called a flat yield curve. There also can be conditions where it is true that short term interest rates can actually be higher than long term rates, although it is not the case in today's market as all rates are low, but it has occurred. The best example I can remember is back in the 1980's when we saw short term rates with a prime rate up around $20 \%$ and long term rates were at double digits but that is actually an inverted yield curve. That was the segment of the market that period of time. Again, if that were the case you would see a yield curve that is actually higher than the short end of the curve than it is on the long end. In addition, and as Robert noted in the narrative there on page 10 the inverted yield curve is generally a negative economic indicator as it is reflecting an implicit assumption that there has to be an economic slowdown and the rates of the future will be lower.

Page 11: Is presenting the treasury yield curve at different points during 2011, and as you can see the darker blue was the shape of the yield curve in January, the lighter blue is the shape of the yield curve and this is for US treasuries in the month of October and the treasury curve in January which is at a higher level it is today. Although it has flattened out a fair amount between the very short rates and the longer-term rates there is still a difference of 300 basis points, so the yield curve may be a bit flatter, but there still is a certain amount of steepness especially in the first 10 years or so. All these factors go into the pricing of bonds that effect pricing and what any investor in the market is willing to pay for a bond of a given maturity on any given day, whether it be through the purchase of bond in a new issue, or working in bond trading in the secondary market, I will now turn to Robert and ask if he walk us through the process of pricing and selling bonds on an ongoing basis.

Page 12: When we start talking about bond pricing we have to talk about a few different concepts, the first being the yield. The yield is connected to price, price and yield are both connected to the coupon and maturity date of the bond that is being sold. Starting with the yield you have to remember that this is the one thing that is sure, and I'm sure most of you who are issuers, cannot control. The yield is given to you by the market and depends on the supply and demand of the bonds and what people feel about the economy and other factors, and it is that interaction of the supply and demand that gives you the yield that you will have to pay as an issuer on bonds to raise money with bonds. Once you have the market supplying you with the yield then we have to go and figure out how that converts to the price of a bond. The price of a bond given the yield depends on the coupon of the bond and the maturity date of the bond. The coupon is the rate of interest that the bond is going to pay to the investor over time. If by chance the coupon that you are buying in the marketplace, is equal to the yield the bond will be selling at $100 \%$ of the value is what we call the underlying value or the intrinsic value, or the value that you will get paid back at maturity, is called a par bond. It is one particular incident in the marketplace that when the coupon and the yield are exactly the same and $\$ 100$ of face value of bond you can buy at $\$ 100$ value price. Of course this only happens once in a while. It does not happen all the time, and the only time you can control this as an issuer is if you are issuing bonds and when you are setting a coupon rate on the bond to match the yield that the marketplace is demanding at the time you sell the bond.

Robert you may want to take a moment and explain that in the initial sale of new bonds such as PRAG described in webinar Bond Math Part One last week, that underwriters and issuers in a negotiated sale decide whether they want to offer the bonds right at par or they think that market conditions are such that investors prefer to receive either a slight discount or premium in the initial sale, but this process can be determined between the underwriters and the issuer in the initial sale. However the secondary market has no control over the pricing at the initial issue sale but they do have some control.

That is certainly the case when you're doing the initial sale is when you get to decide a lot of these aspects. You cannot control the yield but you can control the coupon, the maturity dates, and you can decide if you are selling and what is going to go into the premium or discount bonds.

Starting at this point I am going to ask anybody if you have any questions, please jump in here with one and I will take a look and see if we can answer any of them. I don't see any yet, but if you have any, do not hesitate to ask us to go into something in more detail or slow down as the case may be. I think one message we have is to slow down and a little bit so I think we will try to do that.

We talked about the par bond, so let's talk about one of the hardest things that begin or end in the finance and dealing with bonds that you can have trouble with, and that is the idea of the relationship between the price and the yield. This is something that is always difficult when you're talking to bond traders, even they have trouble making it clear as to what they are talking about in a rising market or falling market. Are they talking about yields rising or prices rising? So it gets very confusing and you have to slow everybody down when are we talking about prices or yields because they move in exactly opposite directions.

Page 13: If you own a bond with a fixed coupon bond and the market yields rise, the price of the bond is going to fall and the opposite happens if the market yield falls the price of the bond will rise in value, so we put together this table for you to give you a feel where the line is the par bond, where the coupon is $5 \%$ and in this case we chose a bond that has five years to maturity and a market yield of $5 \%$ and is selling at 100 or $100 \%$ of its value. If interest rates were to start raising going to $51 / 4$ or 5.50 or $53 / 4$ all the way down to the bottom of the table at a $6 \%$ yield, the bond will be dropping in price as the yield rises and the price will go down all the way down to 95 in our example here. Anytime a bond is selling below its par value it is called a discount bond. If on the other hand, you are lucky enough as an investor to buy a bond at par or let's say and market yields start to fall that means you made a wise investment as you bought at a high-yield, the bond value will go up. In our example is the market yields dropped by a quarter, $43 / 4$, four, the price will go up in value. What this is telling us, is the overall market, if interest rates are falling as they have been certainly in our markets in the last two, three, four months. The value of your investments that you have locked in with these coupon rates that will be paid over a five-year maturity is becoming more and more valuable. People wish they could get the $5 \%$ because now they can only buy $4 \%$ bonds. They will value the bond that you hold; therefore there will be a higher esteem for that bond.

Robert if you get back to slide 13 for one second, just to illustrate a couple things on this page, to comment on these examples we are assuming the bond has a five-year maturity the coupon we are showing here is the official rate of interest is what the issuer will pay over the face value the bond over a five-year period that will never change as that was said at the time the bond was sold. The market yield in this example can literally change every day every minute or every hour. Once the bond is sold to investors and held out in the market, we call this now the secondary market would be changing all the time. The price as Robert showed it changes inversely to the yield, just think of the price as what somebody else is willing to pay for the bond at any given day. As rates go up they will pay less and as rates go up they will pay more but that is as an investor holding the bond how much volatility you actually can have on the value of your investment as it goes on. But the coupon is fixed over the life, the market changes constantly and price changes inversely to the market yield also constantly. Sorry, Robert.

Page 14: Please jump in any time. It's important to keep in mind as you are talking to people and as I said before it can get very confusing because some people talk in terms of yields, some people talk in terms of price, bond traders often times talk in terms of price. So when they say the market is up today that usually means yields are down. They are usually talking the price of bonds have gone up and of course that means the yields have dropped. So when you're talking to somebody and they say that market is up make sure whether they are talking price or yield. The discount bonds that we have been talking about so much, and to give you a feeling for why this has happened, if market yields rise to $6 \%$ your value of the bonds are going to drop and that is because you have the $5 \%$ coupon bonds and that is going to pay the $5 \%$ for the next five years but if interest rates have gone up to $6 \%$ investors are going to want the extra $1 \%$ per year. In a simplified example for five years to get an extra $1 \%$ per year you need to be $95 \%$ of the value of the $100 \%$ you look at maturity. For five years, the $95 \%$ discount to the par value is going to give you about $1 \%$ extra yield a year for the next five years to compensate you as an investor for the fact that you are buying a $5 \%$ on the overall market yields are $6 \%$. There is a question in the market at any given time you have discount bonds, premium bonds, or par bonds that are available for purchase.

## Page 15:

Question: Why would anybody buy a discount bond when you can go out and probably get a similarly rated similarly priced par bond at a $6 \%$ coupon on par bond and get your $6 \%$ yield all the way to maturity? One of the reasons is call protection. Usually municipal bonds and many other corporate bonds and such have the ability of the issuer to call the bonds if yields drop enough that it is worth their while to get rid of the high coupon debt and refinancing and similar to refinancing a mortgage with lower interest debt, lower coupon debt. If as an investor you buy a discount bond which is paying say in our case a $5 \%$ coupon bond but you are buying it at 95 when overall yields are $6 \%$ what that means is interest rates are going to have to fall a lot more in the future for the issuer to get to the point to justify them refinancing your bond. If you are in a shortage of $6 \%$ bonds outstanding and interest rates jumped $5 \%$ you can refinance but if you're $5 \%$ bonds outstanding and a $6 \%$ market environment even if the interest rates dropped from $6 \%$ to $5 \%$ and become par bonds again the issuer still has no incentive to do that refinancing. If you are an investor and you want to guarantee the yield to maturity and not have the bonds fall away from you like to buy discount bonds. On the other side, we have premium bonds. Premium bonds are selling in our case, let's say that interest rates have fallen to $4 \%$ after we have the $5 \%$ bonds outstanding, then the value of the bond has gone up to 105 just like a discount bond which accrues value every year to maturity, a premium bond will decrease, depreciate in value every year to its maturity value because that bond at a $4 \%$ interest rate environment of $5 \%$ bond is paying more than people are demanding so they're willing to pay extra for the bond, take the haircut out of the original price and have it drop every year because when all the dust settles they still are offering $5 \%$. Now, one reason a buyer in the secondary market might decide to buy a premium bond is less volatility and this is a little bit of the trickier concept to convey. The higher coupon bonds do not change in value as much in the market as discount bonds do in the market when interest rates change. When interest rates go up and down on a discount bond on the price tends to swing more wildly while on a premium bond of the price does not swing as much. It just does not move around as much. The reason for this is if you think about the stream of payments you are going to get from a higher coupon bond versus a lower coupon bond more of the value of that bond is in the coupon payments that are going to occur from here to the maturity date which in our example is in five years. You're going to get more of the value of the payments from the coupon stream then in the maturity value of the bond. When interest rates go up and down because the coupon payments are closer to their earlier, in the future, then the final maturity date the present value of these coupon payments are worth more and interest rate changes do not affect the
present value as much on short-term income than they do on long-term income. It is a hard thing to kind of get your fingers around, but if you think about it and think about the stream of payments in the future and what each of the payments they receive what the present value is worth you can begin to get a feel of why a higher coupon getting more payments earlier in the stream of payments is not going to change as much as interest rates bounce around. Sometimes people buy premium bonds because of less volatility some people buy discount bonds because they have called protection built into the bond and they have a better chance of holding that bond for the length of time they want to hold it. Again this is kind of tricky so I want to make sure you have an understanding.

Before we move off this pricing section, both the coupons and yields and also premiums and discounts there was a question submitted about a little confusion on coupons versus yields.

Question: Why people you know why one would prefer to purchase a bond at a premium or discount? Obviously we've touched a little bit on that since the question was submitted but if there is still some confusion or clarification on that if you would not mind resubmitting the question we will try to speak a little bit more to it just to double clarifying that point, the coupon is the rate of interest put on the bond at the time it is issued, it is a legal obligation imposed on the issuer to pay a specific rate of interest on the face amount of the bonds on an annual basis over its life actually through semiannual interest payments. Once the interest is sent in the form of the coupon it never changes over the remaining life of the bond purchase of $\$ 100$ bond with a $5 \%$ coupon somebody will have to pay 250 every six months, five dollars a year in interest regardless of market rates. The yield, like Robert described as the rate the investor, the rate of return the investor would want if they bought the bond in the market anytime after its initial offering. So because the rates could go up or down the price of the bond would also fluctuate. That is the kind of tricky concept we are trying to get across here, but if you want more clarification please let me know.

Question: I will quickly try to go through some of the questions that have been submitted and I I have one that says if you hold on to a bond until maturity and do not sell, your own yield stays fixed, right? That is correct. If you never plan to sell the bond, you buy the bond and plan to hold it until maturity you really don't care what the market does. You're going to own it, hopefully, you'll get your money back when it matures but interest rates could go to $30 \%$ or drop to $1 \%$ and once you buy the bond you're guaranteed to get the investment income that you plan to get as long as you haven't got a Lehman Brothers bond.

Another clarification is and is an absolute truth from the standpoint of the investor holding the bond and is not always true from an accounting standpoint in the sense of unrealized gains and losses. Even in an individual brokerage account nowadays if you buy bonds with a maturity with five or 10 years and you are happy with the return and you never intend to sell them, and my broker still sends me a statement that shows the value of the bonds going up and down over time in a way that could be confusing. And those of you who work on financial statements for your entities might know that you may have to recognize unrealized gains and or losses in your investment portfolios.

Question: Another question is, why do investors care if their bonds are called, what is the risk to them, lower interest rates? Yes that is the risk. If you have a nice juicy $10 \%$ coupon bond and interest rates dropped to say $5 \%$ and the Coca-Cola company that issued the bonds decides to call them away from you and give you your money back and probably even get what is called a premium or maybe even a little bit of extra premium however that depends on the way the bonds were sold initially. But then
you've got this cash that you have to reinvest at $5 \%$ market rate environment and you would dearly love to have kept a $10 \%$ bond, so that is a callable bond.

In that regard and as municipal bonds issuers you should recognize and it is fairly common to have call provisions in the municipal market are actually advantageous to the issuer usually given the right to call your bonds which is usually at par meaning no premium even though they might have initially carried a maturity as long as 30 . So in affect the municipal issuer if the rate environment 10 years from now is more attractive has the right to call the bonds and refinance at a lower rate, whereas the investor who initially bought the 30 year bond doesn't really have a right to turn the bond back in and get a higheryielding bond if the rates have gone up. So in that sense the call provisions in the municipal market are fairly good and the attractiveness is somewhat one-sided to the issuer, but the investor gets 10 years of protection but on a 10 or 20 year bond the investor is still left exposed. In the taxable world including any taxable municipals the call restrictions are more restrictive and less advantageous to the issuer and we will not dwell on this now but felt we should mention it.

Page 16: So far we've been talking about what are called current interest bonds, bonds that have a coupon and pay interest every year usually in our market at least in the municipal market it is semiannual and most treasury bonds it is semi-annual payments, you can pay the interest twice a year. Those are called current interest bonds. There are other more exotic kinds of bonds that you will run across occasionally, there are a lot of technical reasons that people issue them, but let's just describe what they are and how they work. They are called capital appreciation or zero coupon bonds. They don't make any payments, the issuer makes no payments on these until the final maturity date. You buy these bonds, either at a discount, or you buy them at par and they accrue value every year, but they accrue value, but do not actually make any current interest payments, during the year. Mathematically the capital appreciation of bonds is exactly the same. They are sold at a price today. You get paid and accrue value in the future. The only difference between them and it has mostly to do with legal technicalities of what counts as par value for a bond capital appreciation bonds are usually issued at a current par value and everything that accrues from now until they mature is called interest while a zero coupon bond, the final maturity is labeled a par value and you sell it at a discount, so it is telling a discount to par value. Most issuers prefer to issue a capital appreciation bond just for the way the term par-value works into their documents and into various ratios that they have to meet as they do not want to have this very large par value that is maybe five or 10 years into the future and the only cash they have today is a discount of that amount.

Page 17: So it's, let's talk a little bit about what it looks like if you look at two different debt service schedules. As an issuer how you would make payments comparing a current interest bond and a zero coupon bond or a capital appreciation bond also known as a CAB for short and we will avoid trying to getting into initialism's but the table here on this page will show you that if you issue a 10 year bond as an issuer, the first three columns after the year \$500,000 a year in interest for a total payment on years one through nine and in the final year you pay $\$ 10$ million plus the interest $\$ 500,000$ that final year. If as an alternative you decided for whatever reason internal to your special situation as an issuer, you didn't want to make any payments on this bond until the last year you could issue a zero coupon bond. The value today you would get $\$ 10$ million to build the project or whatever it is and do not make any payments until the maturity date and then essentially the accrued interest that has compounded on this bond for 10 years. In our case, they have a 5\% yield if you've got $\$ 10$ million a pay and have a zero coupon bond, your payment in the future would be almost $\$ 16.3$ million. On the very right-hand side we are showing you what the accretion looks like every year even though you are not paying interest,
the value of this bond is going to increase every year because you are promising that at some point in the future and in this case, the 10 years you're going to pay the $5 \%$ rate compounded every year. So if the investor, a mutual fund buys your bond for $\$ 10$ million after year even though they haven't received a payment of the bond is now valued has an intrinsic value of $\$ 10,500,000$. Of course we don't know what's happening in interest rates in the meantime so we do not really know what the actual market value of this bond might be at that time it could be higher or lower or exactly $\$ 10.5$ million, but the value that you have promised to pay back as an issuer to the investor after year one of the $\$ 10,500,000$ accretive value. You do not have to pay it then, you keep accruing this value all the way to maturity until year 10 in which case you do have to write the check and pay the 16.3 million dollars to the investor.

Let me take a quick look to see if there are questions out that because that gets to be a little bit trickier with zero coupon bonds. We have a question.

Question: What would be the benefit because of compounding interest to have a cab that matures in 45 years? While the question I guess is the benefit to whom. For the investor if you are an investor buying a 45 year compounding bond you might be able to lock in an amount of money in investment today. If you're 15 years old so that when you are 60, your retirement could be taken care of, of course depending on what inflation does for the next 45 years but it is a way of deferring income for a long time if you have some specific goal in the future. As an issuer, it gets to be a little trickier. I mean, that joke we used to make about being an issuer to say why don't we issue 30 year bonds as CABs to build this airport and what you do in the 30th year and the joke is of course you default, because you could not possibly make that payment then. Usually what happens as an issuer, you mix in zero coupon or CABs in the overall structure of selling lots of current interest bonds as well. There are sometimes specific places where you could stick a capital appreciation bond, let's say a 10 year CAB and it allows you to structure lots of other current interest bonds earlier and you could issue a lot more, because in effect your debt service you have dug out a whole where don't have to pay interest and you could fill that in with a lot of other maturities which would in an upward sloping yield environment you might be able to lower the overall cost of the entire bond issue because you can issue a lot more bonds at a lower rate. However, it really depends on market conditions and the specific requirements of the issuer about whether they would be interested in issuing CABs or zero coupon bonds. You don't see a lot of them in the market but you do see occasionally people issuing these.

Why don't we keep moving? I think most of the questions Robert has addressed, there is one or two we might need to circle back to. Or maybe question of precedent which may get a little confusing but why don't we keep moving forward to make sure we cover the material and then I will try to keep an eye on questions. I see one quick question about can CABs or coupon bonds be called or refunded? Yes they can, usually you have a schedule that shows you the accretive value like we have on the right-hand side and you can call it either a par which is the accrued value or maybe even some small premium to that amount. So quickly get that question answered.

Page 18 \& 19: On redemptions and accrued interest, when you sell bonds as an issuer you have a coupon date, maturity date, the date that the bonds are scheduled to be paid back, the maturity date. I think last week, if most of you are participating in Bond Math Part One, there was some discussion of serial bonds and term bonds I'm not going to spend an awful lot of time on this. A serial bond is very straightforward as it has a coupon and a specific maturity date. As a buyer you buy the bond and you know when you'll get paid back. As an issuer you sell the bond that you know exactly what the maturity
date is, so it is a very straightforward bond. The issue as a whole maybe composed of a separate number of serial bonds maturing in separately consecutive years. That is correct there could be 20 separate bonds with 20 separate maturities and each one could be a serial bond. Another way to structure the issue would be a term bond. A term bond is where you gather a few of the serial payments that otherwise would have been sold separately, bunch them together and sell them as just one bond with what are called sinking fund payments which otherwise would have been serial payments are actually payments are made on the larger bond. In our example here in this table on page 19 we have a $\$ 10$ million bond with maturity dates in 2012, 13, 14, 15, not the maturity dates but sinking fund payment dates, final payment date is 2015. The reason an issuer you might bunch these together is really just more for convenience to sell larger groups of bonds, if you have a bond issuer with lots of tiny different serial payments instead of selling all those separately and having to keep track and manage them overtime you sell one term bond, bunched up together. You might get a different price, quite often you do because larger bonds even the sinking fund payments the bigger bunches are easier in the market for trading among larger investors, mutual fund companies, insurance investors, if they decide at some point in the future to sell they can find a seller. I do not want to go too much into this concept because I think we've gone into this at least once to some extent.

Page 20 \& 21: The last topic in this section, bond redemptions and accrued interest on page 20 , as we've seen and when you get to the maturity date as an issuer you've got to pay back the bonds. Usually when involved with the issuing of bonds there is a paying agent who takes care of the nitty-gritty details of collecting the money from the issuer and making sure that the bond holders get paid. If you are a big enough issuer like I'm sure some of you are in California you can be your own paying agent where you take care of those details that you don't have to have an outside person doing it. On the day you pay back the bonds including all accrued interest on the bonds as every day the bond is outstanding it accrues interest. Anytime you call a bond even in the middle of the month or on an odd date you have to pay interest on the bond through the date in which you're going to pay off the bond. At maturity usually you owe the last six month payment, which is the interest payments on the bond. Then we threw in a little humor what is the difference between a bond and a bond trader, a bond actually matures. If any of you have dealt with bond traders which I assume some of you have got to know them enough that this is a pretty accurate statement, there.

Question: There was one other question I notice that we ought to touch upon is there a lot of demand and CAB bonds? The short answer is in the current market, no. There have been times in the market when they are quite popular with investors and because they're popular with investors they can offer automatic advantages to issuers. We haven't seen those conditions and a well so they are really included in bond deals in the last several years We talked about CABs and cutting out there are even some more exotic bonds which I won't go into any detail, just to show that the universe of bonds is actually greater than what you have seen so far. If you want to find out more about them, just send me an e-mail or give me a call and I will be happy to talk about it.

Page 22: This section, we are going to go through some of the built-in formulas, the functions actually that are available in Excel that you can use to do your calculations of price yield, internal rate of return and then a payment function and I will go into each of these in some detail. It is nice to know that each of the functions are available and that in the market quite often you can have underwriters or people who are proposing things to you and say this is a price for a yield that we think will pay you for a bond that you have, or that we think you should issue. But it is good to be able to have at least some idea of
how you could do your calculations to double check what they are doing or to figure out for your own internal purposes what the value of things are and what the market is doing. So let me go through these.

Pages 23 \& 24: The first one, which is a very, very handy function in Excel is the price formula. It actually calculates the price of a bond when you give it all the details of the different parameters that you have to go into the formula. It looks kind of complicated but it is really not that complicated so even though you see a lot of words on the page and it looks like it's got all sorts of mumbo-jumbo here if we just go through it I think you'll see it is pretty straightforward. If you dealt with functions in Excel, you know you plug-in = and put the name of the function that you have to give and then you have to give different arguments for the function. In this case, we have to give the price formula all the information it needs to figure out what the price of your bond is going to be. It could be a treasury bond, could be a municipal bond, could be corporate bond, Fannie Mae, Freddie Mac security, agency bond, but from my point of view since I am a municipal bond person all my examples will be oriented toward the value of a municipal bond. So let's talk about the different pieces that go into the price formula. The first is the delivery date, which is the date that you will exchange the bond for the cash being paid for it. That's the date when you will trade the bond for the money. The maturity date also straightforward as the next item that goes into the formula. The coupon, which again is that rate that is guaranteed to be paid on the bond, doesn't depend on the market which is the next item, the yield. The next item is the yield and that is the end of the market is going to tell you what the value of that is or perhaps the person who wants to buy the bond will say I demand a $5 \%$ yield for your $6 \%$ coupon bond. So you put that in there. The value at maturity, they put this in there to make sure the formula, the function is more flexible, it's almost always good to be 100. It's unusual to have bonds at least in our market that would have a maturity value different from par value. So in our case it's almost always 100. The number of coupons payments per year is almost always to, the Treasury bond, municipal bonds almost always have two coupon payments a year. And at the day count basis there are different ways in different securities how you count the number of days in a year. For treasuries is 365 actual numbers of days. In our market is 360 day year with 30 day months. There is a whole history that goes back to why that is the case. So for the purposes of Excel, the day count bases you can either put in a zero and leave it blank of zero means 360 day year, and I think it is one is a treasury, the actual day here. Just to clarify one point on that, Robert, these arguments, when you use the price function in Excel the arguments need to be entered in the sequence you identified on the page. Yes, that's correct; it needs to be in that order, Beyond that the person using the function does not have to write formulas or do anything, they just enter the price function and enter the data in the correct order. Yes that is exactly correct

Pages 25-28: So here is the example problem. You're being asked if you're interested in buying a municipal bond, you are not told what the price of the bond is but you are told that the yield is 450, $4.50 \%$. The coupon is $5 \%$, the maturity date is January $1^{\text {st }}, 2030$, and the delivery date is September 19, 2011. How do you use this information to plug it into this formula? Here is the actual, I plugged this in and created this in the spreadsheet and took a picture of it to insert it in this slide so that you can see the actual values here.

Now, the formula where I have the equal price in cell B-3 shows that the arguments are 7 through B 13 separated by commas. I did that to make it very easy if you want to change this in the future, if you want to change the delivery date, change the coupon change the yield or maturity date, this is a very flexible way to create a little formula and you can copy this exactly and build it into a spreadsheet, pull it up anytime you want to plug in delivery date maturity date coupons and yield and you can look at the result of what that bond is, this is for municipal bonds so that they count base is zero, 360 day year. And

I think if you just want to copy this and plug it in you would have an easy little place to always do a calculation of a bond price. The opposite part of a price is calculating a yield. If somebody comes to you and says I have a bond that I'm going to sell to you I would like to sell to you at 103, will you buy it? You have to figure out what is the yield on that bond so you can compare to the market and see if you're getting an appropriate deal or if somebody is time to take advantage of you. The yield is really the opposite calculation from the price. It takes almost exactly the same arguments, it has basically the same structure, but instead of popping out a price it will pop out a yield. The arguments for the yield formula are delivery date just like before, maturity date, coupon, of the same components you had before the price formula, the difference here is we don't know what the yield is that we are given a price we plug in a price next in the yield calculation, value to maturity is 100, to payments here 360 day year. So as an example problem compared to the previous one this time you are offered a municipal bond you are told the price but not the yield, you are told the price of the bond is one of 106.179, delivery date is the same date as before, September 19, the maturity date is January $1^{\text {st }} 2030$, how do you calculate the yield?, The price, which is the variable we are playing with the value of maturity and the coupon payments per year. So I know we are going to this pretty quickly. I just want to make sure we try and get through everything.

Question: I see one question they say one equals 360, the year... no, the assumption for the day Basis of zero is that 360 day here in Excel one is the actual day and I believe there may actually be a two which is the actual over 360 which I don't want to get into. There are other securities that do calculations in different ways that mix of both the 360 day and actual way of calculating things, but that's beyond what we want to get into here. For municipal bonds remember zero, for treasuries remember one.

Page 29: IRR. The internal rate of return, this is probably, this is one of the more complicated concepts and the way you implement it as an issuer of bonds is going to be a little trickier so you have to bear with me a little bit. It looks like a fairly simple formula. And it doesn't take many arguments, but getting to the point where you can actually use the IRR formula is the hard part that is where you have to put in quite a few details of the bond issue. To get to your IRR.

The IRR in my world of municipal finance is really a true interest cost escalation. As an issuer you are trying to figure out with issuance of bonds with one or maturity at $2 \%$ some with 20 or maturity at $2 \%$. And what you want to do is figure out what is the overall cost of funds for my bond issue. I am building my library. I want to know what is the yield, what am I having to pay overall blending all these rates together to build my library. But you need to do is calculate the internal rate of return which is as a definition is just that interest-rate, that when used to discount all the future cash flows out to the value I'm getting today, the value of getting from my bond issue equates to that value to the present value of all these future payments we are going to be making. I assume most of you are finance people so you understand what an IRR is. But that is the basic idea, it is overall rate of return and in our world it is called the true interest cost. In kind of a slightly simpler way to state it, it is the rate which, when used to discount the stream of future debt service payments on the issue back to the delivery date is equal to the net proceeds of the issue. It is taking into account both all the interest rates on every maturity of the bond issue and also taking into account the cost of issuing the bonds and bringing it all together to get to the overall true interest cost is an issue which as Robert says is an internal rate of return calculation.

Page 30: On slide 30 here is an oversimplified example that will still be a little bit involved, stick with me this is probably going to be the most involved thing we will talk about it the entire presentation, here but in order to explain the IRR we have to go through the steps because there really is no other
simpler way to do this that I could come up with, anyway. Here's the example we are going to use. We will assume a simplified bond issue with just two maturities. Most bond issues will have between 20 and 25, maybe 30 maturities. In our example, we will use a bond with two maturities. In the spreadsheet we enter the cash flows to be paid on each of the bonds that have been sold. We add up the payments so that we know the total amount that we are going to pay each year with these two maturities on our bond issue. We are going to calculate how much money they receive from the sale of the bond issue and then we'll calculate the internal rate of return, the true interest cost on the bond issue. We are going to assume the bonds are sold on January 1, 2012. The first bond matures on January 1, 2014 has coupon of $4 \%$ a yield of $41 / 4$ and because we know the yield we know the price, which is 99.525 when we plug into the spreadsheet formula. As par value of $\$ 500,000$ result a $4 \%$ coupon, 400 , with $41 / 4$ yield. The second maturity is one year later, January 1 , 2015 we assume a coupon of $43 / 4$, a yield of 450 a little bit longer maturity assuming an upwardly sloping curve, little bit higher rate, has a price of 106.94, and we are going to assume a slightly larger price, $\$ 750,000$ as part of the true interest cost of calculation which involves taking money that we don't actually get, when we sell bonds we have to pay lawyers and underwriters and I am very happy to say financial advisors, too. We are assuming it's about $1 \%$ of the value of the bonds so that is money paid to other people, you never see it but that has to go into the calculation as it is the cost of issuing the bonds. And I do not want to scare anybody out here but this is the spreadsheet that I put together and it might be a little hard to read but you can calculate the internal rate of return, or IRR. The second box and what we call the bond details shows the parameters that we put into the two maturities we mentioned in the previous example. The first maturity January $1^{\text {st }} 2014$ and the second maturity, January $1^{\text {st }} 2015$, with a coupon rate of $4.75 \%$, a yield of $4.50 \%$ and a par value of $\$ 750,000$. Plug in the coupon yield and then the price behind that cell and the prices just the price formula that we saw earlier grabbing the maturity date, settlement date, coupon yield and calculate the price. I then multiply the price times the amount of the bonds to come up with a value received from the investor, the person who actually buys the bonds will pay the $\$ 497,000$ plus $\$ 750,000$ for the next bond. Unfortunately as the issuer you don't get to keep all that, $1 \%$ goes to the other people involved in the transaction giving you a net price. If you add up the two maturities that you sold of $\$ 1,240,000$. So you are selling $\$ 1,250,000$ bonds, one bond at $4 \%$, and one bond at $43 / 4$ and you're getting $1,240,000$ from the proceeds of the bond.

Page 31: The next section is how do we calculate the total payments we are going to have to make for the next four years on these bonds now that you have sold them. The first bond maturity has $\$ 500,000$, 4\%. You're going to pay, again, and to be a little careful here about is we are talking about semiannual payments here, so we are trying to be very accurate about what the payments are we are not just grouping together annually. But semiannual payments July 1st and January 1st are the payments dates, so it is $\$ 10,000$ each semi-annual payment and $\$ 20,000$ year on the first bond, and $\$ 750,000$ for $43 / 4$ for the second bond we sold so it is about $\$ 35,000$ a year, $\$ 17,800.00$ for a semiannual period. Once you figure out the two payment streams you add them together and say what are the total checks I have to write each six months now but I borrowed this $\$ 1,250,000.00$ and the total cash flow shows you what the payments are. July $1^{\text {st }}, \&$ January $1^{\text {st }} \$ 27,813$ up until we get to the maturity date of January 1 , 2014 where it is $\$ 527,000$ we have to pay back the maturity value of that first bond. Now that the bond has gone your interest rate drops so you are only left with the $\$ 27,813$ payment for the next sixmonth period and finally when you paid off all your bonds it is $\$ 67,800.00$. The last step in the IRR calculation is to plug this into a spreadsheet and then grab the stream of cash flows, the total cash flows and put that at the top, the amount of cash that you receive when you sold the bonds. That negative amount you enter in in this case in cell C-37, the top of the column and one of the painful parts of the IRR calculations in Excel is you have to have all the cash flows lined up together because
you are going to aim at those cash flows in order for Excel to calculate what the IRR is on that stream of cash flows. Another thing to keep in mind is the number at the top has to be negative. Or if it is positive the other cash flows have to be negative. One or the other of the group has to be a negative amount, you can't have it all positive, and Excel would not know what to do if you made them all negative for all positive. I think one way to look at it is, the first cash flow is really looking at it from the investor's point of view is I am spending my $\$ 1,240,000$ and I'm receiving $\$ 27,800$ for the first period of income in the future with others to come in the future and what is the overall yield. Just make sure either the first number is negative, or the other stream is negative but they can't always be the same sign. It's a little tricky however, if you want to go over that in more detail please call or write me about that. If you go all the way back up to the top you will see that big based on these facts it was put together basic example the overall rate of return is $4.836 \%$ but true interest cost, the IRR is $4.836 \%$. One small caveat to note over to the side is everything is done semi-annually in the municipal bond market we have to double the IRR calculation to convert it to an annual rate. The straight IRR calculation gives you a final rate in this example so this tells you that your overall costs of funds in our example are going to be $4.836 \%$. I will take a breath and look at the questions here and see if there are any that I can answer.

Question: When you enter dates in the Excel functions, Robert, you enter them as numbers, you might want to clarify that. Answer: In the price and yield formula for a maturity date and settlement date you have to enter it in the way Excel count days. So there is actually another function called the date function which has as its arguments a month, day and a year. So you've got to first when you are plugging in the settlement date or the maturity date in the formula you actually have to embed another function and cited called date, date open parenthesis and if I remember year, month, date I might have to double check that I could kind of automatically. And close parenthesis. So you have to put date, drop that in the place that the yield of the price formulas, the arguments that they are taking. So be a little careful with that one.

Question: The guess portion of the IRR. The guess portion has to do with that nature of what an IRR does, or the way computers work. There can be multiple solutions to IRR calculations if your cash flow pops up and is very negative more than once. As I was saying you got to have one negative and a whole bunch of positive and that will give you a unique IRR solution. That is just one solution to that group. Some cash flows start bouncing around I will not get into it but that could give you multiple IRR solutions are equally valid. So the IRR calculation has to start with a guess the best way to guess as to use the average yield on your bond issue. That will get you very close to what the overall true interest cost is going to be in the guess is really just a way for Excel to have a starting point. Computers are not start smart enough to look at it and say oh let's start around 5 so I will start guessing at 5 so you have to do a little bit of thinking for them. If you leave the cell blank Excel will assume a rate of $10 \%$ as the guess. That is fine but may be too far off from what the true guess actually is in Excel, it will give you an error as it goes through a number of iterations to calculate what the IRR is. I think it is something like 100 iterations and if it does not converge on an answer fast enough it will give you an error so it's best to give a guess somewhere in the realm of possibility of what the true interest cost is going to be. If you were doing that on a bond yield the easiest guess to use for example would simply be the highest coupon rate, bond yield is going to be in the right ballpark for sure.

Question: A few people asked why I use . 045 as the guess but that was just me plugging in a rate that is in the realm of possibility for my guess. Robert, you do have a typo in the table you show both bond maturities, the table to the right is the second, but it is just a guess of going to build something worth
the cost.

Page 32: Let's go to the example here, a small bill is considering a proposal to build a new library to cost $\$ 10$ million wants to make a general estimate of what the annual debt service will be, so assume the bond will have 30 year maturity the average bound rate will be $5 \%$, and the annual payments on the bonds will be level over the life of the bond issue. So you have a 30 year bond, average rate we think will be around $5 \%$, what is our 10 million dollar library going to cost us a year for 30 years. Actually you are doing this as a $\$ 10$ million bond issue, if it is a $\$ 10$ million library we not adding up the cost of issuance or the reserve fund or things that PRAG may have talked about last week in Bond Math One webinar series this is just identifying the debt service on a $\$ 10$ million bond deal. For this example we are just trying to Guesstimate a quick and dirty way without having to create an involved spreadsheet as we are trying to get a spreadsheet, a quick idea so you can tell the mayor what the library will cost. So the formula takes the interest rate first, the years to maturity, the bond size or the cost, 10 million, the result that pops out of 650 or a little over $\$ 650,000$ so to build the library and paid back over 30 years you need to be prepared to pay $\$ 650,000$ a year. So that is a good way to end the formula section to enter the formula section because it is a lot simpler than the crazy IRR one.

Question: We have one question about the difference between IRR and MIRR and XIRR which are also other formulas in Excel. They are a little more technical. The modified internal revenue of return, and I think it will cause everybody's eyes to glaze over if I go into detail but whoever's asking the question if you give me a call or send an e-mail I will be happy to go over it with you.

Question: Someone has asked about doing some treasury calculations. And there will be happy to do that also. I would again suggest, give me a call and we can go over that in some detail.

Question: In that same realm, Robert had a question in that realm earlier in the duty of 5\% coupon and the yield went to 4 why wouldn't the price of the bond go to 125 as opposed to the simple example of 105. It may be easier to deal with that question one on one and go through the math a little slower because intuitively one is thinking the price should change by the same ratio as the yield to it but obviously it does not work quite that way. So if you want to go into more detail you can either contact Robert by email or phone.

Question: Whoever asked if we will give you contact information at the end of this webinar, Robert is definitely the better person to talk to and weigh in on the technical aspects. He will be able to clarify that pretty quickly for you. I believe that is the end of the questions for the formula session.

Question: This last question asks if you use treasuries how do you calculate the price, do you see that? Yes it is basically the same formula, but you plug in a one for the day count basis instead of zero, so the formula should work in that case. It gets a little trickier if you are dealing with things like T-bills. Most of them have to do with bonds, usually more than a year to maturity of your dealing with shorter-term maturities there are some other built-in functions that deal with those so you have to be a little careful with those again we are talking mostly of his presentation about long-term bonds instead of the nittygritty of all the different price calculations but I will be happy to go over some of the other functions that are available. Excel has a whole group of financial functions to use. So I just touched four, there are probably 15 other financial functions for other situations, so I encourage you to go into the help section, call me or go to the help section of Excel and go through the whole finance section of functions because there are lots of handy ones in there.

I think the next page or the following as the contact information let's make sure that people see that, here.

Linda or anyone at CDI AC are there any other questions you are seeing you would like us to address?
Question: I see one just asking about the payment function, why didn't we assume semiannual interest payments and would make a tiny difference? You could take the average bond rate divided by two and take 30 years of maturity of multiply by two to get 60 semiannual payments but it's hardly going to make any difference to the ultimate result. So that is the reason, just make it simpler. And if you are calculating a payment on a bond, you really, you don't have level annual payments, rather semiannual payments because principal payments typically get paid once a year so you have usually semiannual interest payments and annual principal is a more precise calculations so whichever way you code the payment function to get an approximation of the way the real number works and it will be almost identical.

Question: How do you deal with determining the bonds size amount? We did not go into that. It was covered I think in great detail in the first session with PRAG. They covered how you calculate things like interest in the reserve fund, bond sizing up an issuer when you want to build a project. We did not go into that a little but it was covered in great detail by the first session. I think we have answered all of the questions that have been asked and we enjoyed doing this and putting this presentation together.

Robert, this is Mark again with CDIAC. We've had a little technical problem so I want to close out the webinar. Thank you both for an excellent presentation. I want to make sure we recognize our future webinar programs available on the CDI AC website at www.treasurer.ca.gov/CDIAC I also want to recognize the other classroom-based training also available at the same website. Thanks to the CDIAC unit Linda, Margaret and of course Marshall and we look forward to your future participation. I understand that we were able to get responses to questions that we did not address in the first webinar and we will post them to the website. We can follow up with the speakers to do the same thing addressing questions that they did not get to. As well, their contact information is available in the slides. So, follow-up seems to be encouraged by both speakers and we thank them again.

With that, we look forward to your participation in future webinars.

