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CALIFORNIA DEBT AND INVESTMENT ADVISORY COMMISSION

BOND CASH FLOWS LITERACY

INTERMEDIATE BOND MATH (PART 1)

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August 7, 2014 2:00 PM – 3:30 PM

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CALIFORNIA
DEBT AND
INVESTMENT
ADVISORY
COMMISSION

BOND CASH FLOWS LITERACY

INTERMEDIATE BOND MATH (PART 1)

PRESENTED BY LOUIS CHOI
PUBLIC RESOURCES ADVISORY GROUP
AN INDEPENDENT REGISTERED MUNICIPAL ADVISOR (IRMA)

Topics

- Bonds and Loans
- How Municipal Bonds are Priced (or Valued)
- Understanding Cash Flow Schedules
- Debt Amortization
- Bonus: Using Microsoft Excel Functions

Bonds and Loans

Bond cash flows literacy

intermediate bond math (Part 1)

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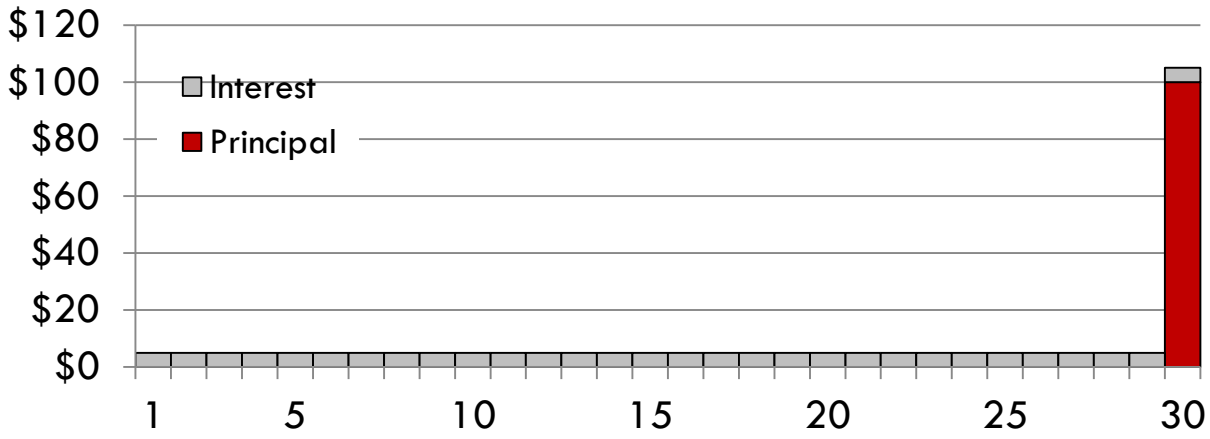
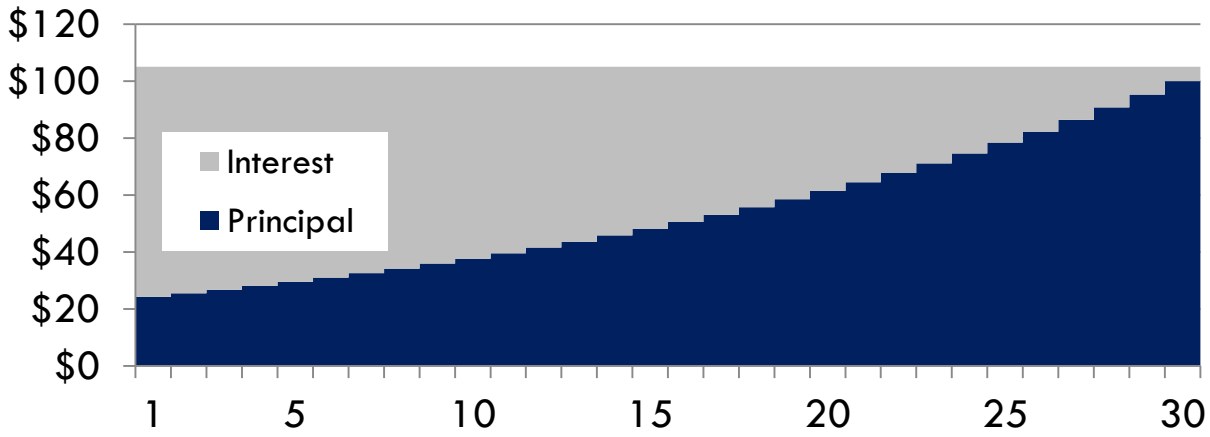
presented by: PRAG

Bonds as Loans

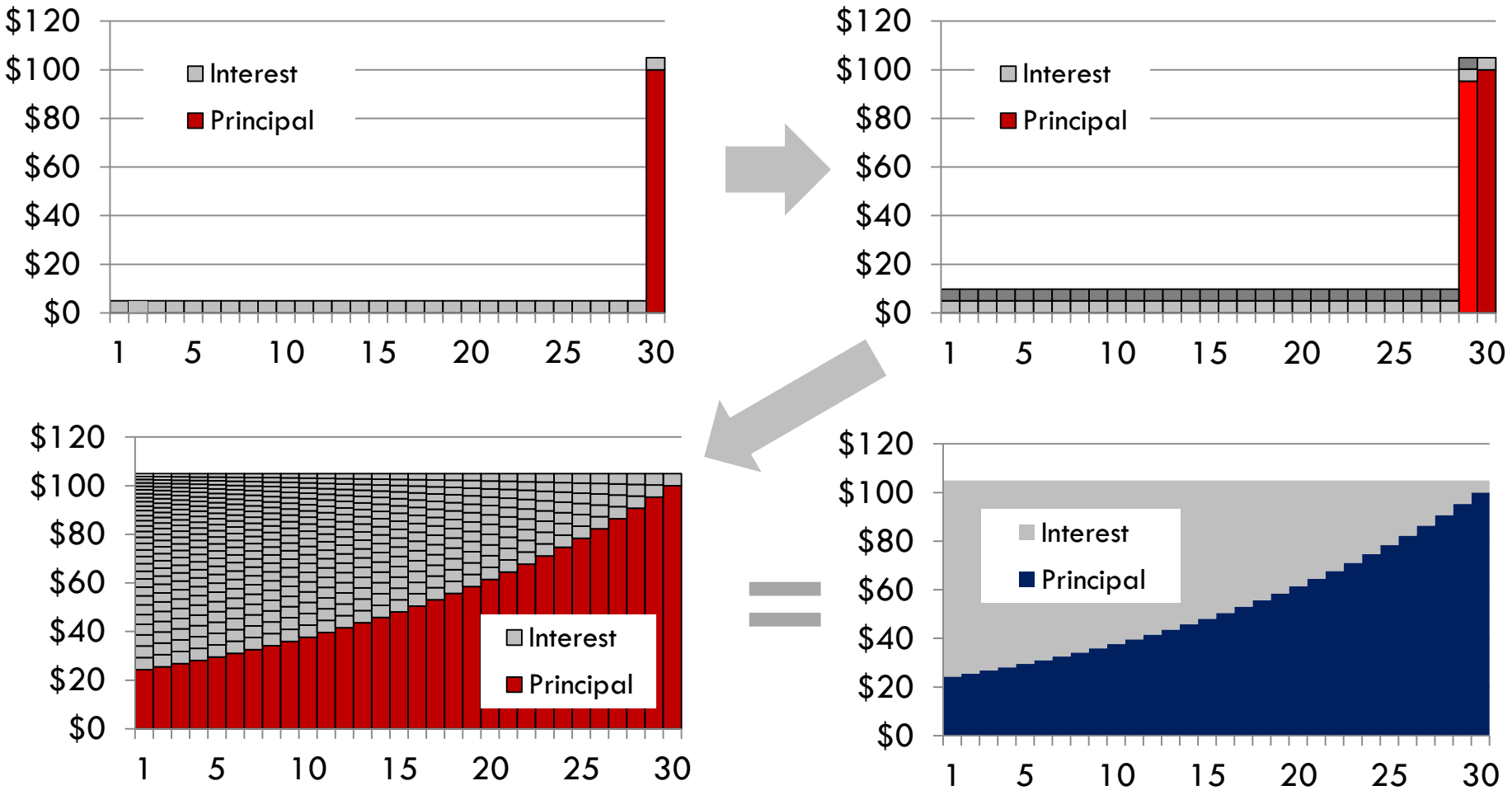
An investment…



A loan…



In Aggregate, Bonds in an Issue Are Equivalent to a Loan



A Bond Issue and a Loan Are Mathematically Similar, But Not Identical

A Loan:

Date	Principal Balance	Principal	5.00% Interest	Debt Service
5/1/2014	50,000,000			
5/1/2015	50,000,000	9,050,000	2,500,000	11,550,000
5/1/2016	40,950,000	9,500,000	2,047,500	11,547,500
5/1/2017	31,450,000	9,975,000	1,572,500	11,547,500
5/1/2018	21,475,000	10,475,000	1,073,750	11,548,750
5/1/2019	11,000,000	11,000,000	550,000	11,550,000
Total		50,000,000	7,743,750	57,743,750

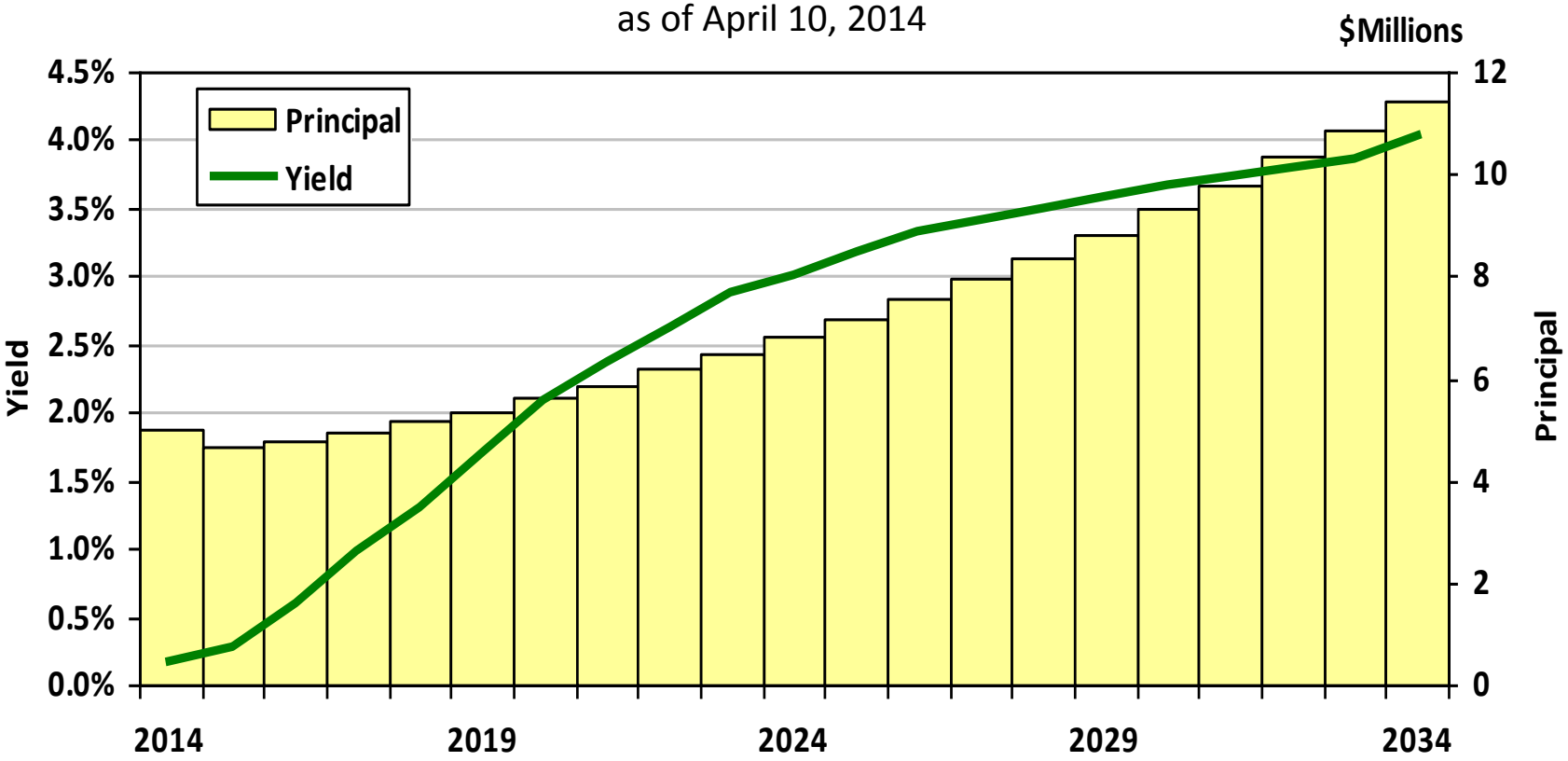
A Bond Issue:

				2.00%	3.00%	4.00%	4.50%	5.00%		
	Principal			Interest on Principal Due					Total	Debt
Date	Balance	Principal	Coupon	5/1/2015	5/1/2016	5/1/2017	5/1/2018	5/1/2019	Interest	Service
5/1/2014	50,000,000									
5/1/2015	50,000,000	9,050,000	2.00%	181,000	285,000	399,000	471,375	550,000	1,886,375	10,936,375
5/1/2016	40,950,000	9,500,000	3.00%		285,000	399,000	471,375	550,000	1,705,375	11,205,375
5/1/2017	31,450,000	9,975,000	4.00%			399,000	471,375	550,000	1,420,375	11,395,375
5/1/2018	21,475,000	10,475,000	4.50%				471,375	550,000	1,021,375	11,496,375
5/1/2019	11,000,000	11,000,000	5.00%					550,000	550,000	11,550,000
Total		50,000,000		181,000	570,000	1,197,000	1,885,500	2,750,000	6,583,500	56,583,500

Differences in Rates Across Maturities Generate a Yield Curve

State Public Works Board of the State of California
$152,420,000 Lease Revenue Bonds
(Department of Corrections and Rehabilitation), 2014 Series C

Principal Amounts and Initial Reoffering Yields
as of April 10, 2014



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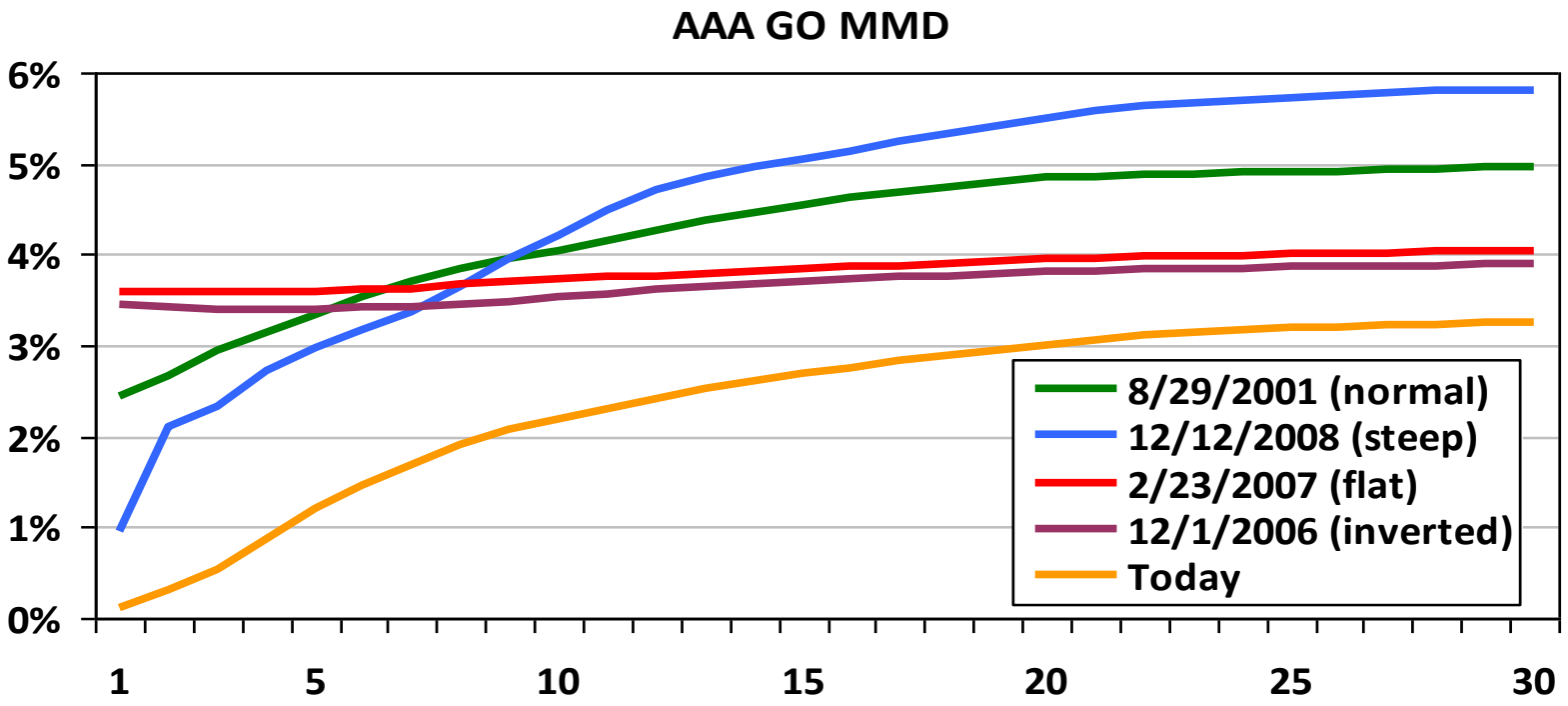
Selected Historical Yield Curves



Percentile/Date	25th	50th	75th	8/29/01	12/12/08	2/23/07	12/1/06
1Y vs. 5Y MMD	0.52%	0.88%	1.30%	0.88%	2.01%	0.00%	–0.05%
1Y vs. 10Y MMD	0.95%	1.55%	2.35%	1.59%	3.26%	0.12%	0.08%
1Y vs. 30Y MMD	1.65%	2.50%	3.44%	2.50%	4.86%	0.44%	0.44%

Calculating Bond Prices

Bond cash flows literacy

intermediate bond math (Part 1)

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Time-Value of Money (TVM)

Calculates the value of future-day dollars in present-day dollars, and applicable to calculations for:

- *Opportunity cost*
- *Inflation*
- *Investments*

- Present Value Formula:

$$PV = \frac{FV}{\left(1 + \frac{i}{p}\right)^{t}}$$

- "PV" = Present Value
- "FV" = Future Cash Flows
- " i " = Interest Rate
- " p " = Compounding Periods Per Year
- " t " = Time or Periods

TVM Is the Basis for Calculating Bond Prices

A stream of future cash flows, such as the periodic payment of interest and final payment of principal, follows the same approach as the sum of multiple terms

- Present Value Formula for Multiple Future Cash Flows:

$$PV = \frac{CF_1}{\left(1+\frac{i}{p}\right)^{t_1}} + \frac{CF_2}{\left(1+\frac{i}{p}\right)^{t_2}} + \ldots + \frac{CF_n}{\left(1+\frac{i}{p}\right)^{t_n}}$$

  - "PV" = Present Value, or Price
  - "CF" = Future Cash Flows, which for bonds include:
    - ✓ Principal
    - ✓ Semi-Annual Interest
  - " i " = Interest Rate, or Yield
  - " p " = Compounding Periods Per Year
    - ✓ (Municipal Convention = 2)
  - " t " = Time or Periods
    - ✓ (Municipal Convention = $\frac{\frac{30}{360}Days}{180}$)

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Bond Pricing Formula

- Municipal Standard Price Formula:

$$P = \left[\frac{RV}{\left(1+\frac{Y}{2}\right)^{\wedge\left(N-1+\frac{E-A}{E}\right)}}\right] + \left[\sum_{k=1}^{N}\frac{100*\frac{R}{2}}{\left(1+\frac{Y}{2}\right)^{\wedge\left(k-1+\frac{E-A}{E}\right)}}\right] - \left[100*\frac{A}{B}*R\right]$$

Principal Component | **Interest Component** | **Accrued Interest Component**

Expressed as a percentage of the principal amount

- "A" = 30/360 days from dated date to settlement date
- "B" = Days in the year (usually 360)
- "E" = Days in semi-annual period (usually 180)
- "N" = Interest payments between settlement and redemption dates
- "P" = Dollar price (as a %)
- "R" = Annual coupon (as decimal)
- "RV" = Redemption value, including premiums, if any
- "Y" = Yield (as decimal)

Prices Can Vary Greatly with Different Coupons and Maturities

- 10-year bond with a 3% coupon at yield of 3.15%

$$\left[\frac{100}{\left(1+\frac{3.15\%}{2}\right)^{\left(20-1+\frac{180-0}{180}\right)}}\right]+\left[\sum_{k=1}^{20}\frac{100*\frac{3\%}{2}}{\left(1+\frac{3.15\%}{2}\right)^{\left(k-1+\frac{180-0}{180}\right)}}\right]-\left[100*\frac{0}{360}*3\%\right]=98.721$$

- 10-year bond with a 5% coupon at yield of 3.15%

$$\left[\frac{100}{\left(1+\frac{3.15\%}{2}\right)^{\left(20-1+\frac{180-0}{180}\right)}}\right]+\left[\sum_{k=1}^{20}\frac{100*\frac{5\%}{2}}{\left(1+\frac{3.15\%}{2}\right)^{\left(k-1+\frac{180-0}{180}\right)}}\right]-\left[100*\frac{0}{360}*5\%\right]=115.764$$

- 10-year bond with a 5% coupon at yield of 5.165%

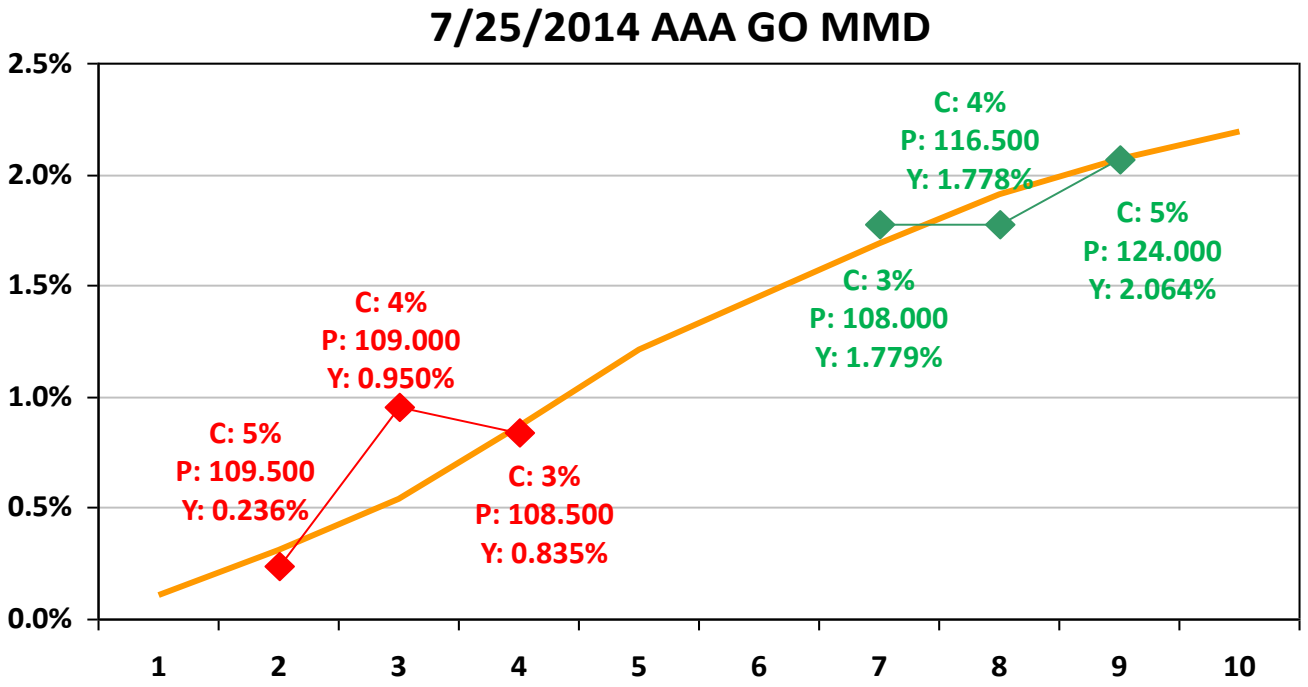
$$\left[\frac{100}{\left(1+\frac{5.165\%}{2}\right)^{\left(20-1+\frac{180-0}{180}\right)}}\right]+\left[\sum_{k=1}^{20}\frac{100*\frac{5\%}{2}}{\left(1+\frac{5.165\%}{2}\right)^{\left(k-1+\frac{180-0}{180}\right)}}\right]-\left[100*\frac{0}{360}*5\%\right]\approx 98.721$$

Terminology:

- *Par: Price = 100*
- *Discount: Price < 100*
- *Premium: Price > 100*

Bond Prices are Commonly Expressed in Yields for Ease of Comparison

Yields help to inform consistency of pricing as terms vary



	Yield >>	0.11%	0.31%	0.54%	0.87%	1.21%	1.45%	1.69%	1.91%	2.07%	2.19%
Coupon	3.00%	102.887	105.359	107.310	108.355	108.659	108.876	108.614	108.050	107.600	107.239
	4.00%	103.886	107.351	110.282	112.278	113.496	114.602	115.189	115.437	115.773	116.176
	5.00%	104.885	109.343	113.254	116.201	118.334	120.329	121.765	122.823	123.946	125.113

Bond Pricing Conventions

Using the price formula when coupon equals yield may result in a calculated price of 99.998 or 99.999

Guarantees investors that the stated yield would be achieved, regardless of whether or when the issuer exercises its option

Prices do not have to calculated for every date; instead, only first dates when redemption prices change must be checked

- Bonds where coupon equals yield are priced at 100.000 (or par)
- Prices are truncated to third place after decimal
  - *Ex.: price of 107.186243… becomes 107.186*
  - *Ex.: price of 98.53293… becomes 98.532*
- Yields are rounded to the nearest third place after decimal
  - *Ex.: yield of 5.16435…% becomes 5.164%*
  - *Ex.: yield of 3.18987…% becomes 3.190%*
- For optionally callable premium bonds (i.e., coupon > yield), bonds are priced to that call date which results in the *lowest* price
  - *Ex.: 11/1/2028 maturity, 4.2% coupon, 3.15% yield, callable on 11/1/2024 at 102, on 11/1/2025 at 101 and on 11/1/2026 at 100, and settled on 11/1/2014*

Assumed Redemption Date	No. of interest periods (N)	Redemption value (RV)	Price (P)
11/1/2024	20	102	110.410
11/1/2025	22	101	110.406
11/1/2026	24	100	110.424
11/1/2028	28	100	111.813

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Capital Appreciation Bonds (CAB)

Also based on TVM formula

- Interest is compounded and paid at maturity
  - Growth in value of a CAB is expressed as an accreted value

$$AV_n = PR \times \left[1 + \frac{Y}{2}\right]^{\wedge(n \times 2)}$$

Note: Prices may be expressed as percentage of delivery date principal amount or final maturity amount, depending on how issuance principal is expressed

  - "AV_n" = Accreted value at period n
  - "PR" = Initial price (generally par)
  - "Y" = Yield
- Generally, not subject to optional redemption
- Sold in denominations such that the final accreted value of each denomination is $5,000

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Capital Appreciation Bonds (Cont'd)

- Solving for an accretion table…

$$AV_n = \frac{DN}{\left[1 + \frac{Y}{2}\right]^{\wedge((M-n)\times 2)}}$$

  - "AV_n" = Accreted value at period n
  - "DN" = Accreted value at maturity (effective denomination)
  - "Y" = Yield
  - "M" = Maturity

Example:
Delivery Date: 5/14/2014
Maturity: 5/1/2019
Yield: 3.50%
Effective Denomination: $5,000

Date	Accreted Value
5/14/2014	$4,208.91
11/1/2014	4,277.21
5/1/2015	4,352.06
11/1/2015	4,428.22
5/1/2016	4,505.71
11/1/2016	4,584.56
5/1/2017	4,664.79
11/1/2017	4,746.43
5/1/2018	4,829.49
11/1/2018	4,914.00
5/1/2019	5,000.00

Cash Flow Schedules

Bond cash flows literacy

intermediate bond math (Part 1)

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Describing a Bond Issue with Numbers

Goal: to understand how the numbers that describe individual bonds and a bond issue work

MATURITIES, PRINCIPAL AMOUNTS, INTEREST RATES, YIELDS AND CUSIPS

$28,790,000 2012B Serial Bonds

Maturity Date (June 1)	Principal Amount	Interest Rate	Yield	CUSIP†	Maturity Date (June 1)	Principal Amount	Interest Rate	Yield	CUSIP†
2015	$1,075,000	4.000%	0.830%	1306853D0	2024	$1,575,000	5.000%	3.010%C	1306853N8
2016	1,120,000	4.000	1.090	1306853E8	2025	1,655,000	5.000	3.200C	1306853P3
2017	1,165,000	4.000	1.350	1306853F5	2026	1,735,000	5.000	3.320C	1306853Q1
2018	1,210,000	4.000	1.620	1306853G3	2027	1,825,000	5.000	3.410C	1306853R9
2019	1,255,000	4.000	1.890	1306853H1	2028	1,915,000	5.000	3.490C	1306853S7
2020	1,310,000	4.000	2.170	1306853J7	2029	2,010,000	5.000	3.570C	1306853T5
2021	1,360,000	5.000	2.400	1306853K4	2030	2,110,000	5.000	3.640C	1306853U2
2022	1,430,000	5.000	2.570	1306853L2	2031	2,215,000	5.000	3.700C	1306853V0
2023	1,500,000	5.000	2.770C	1306853M0	2032	2,325,000	5.000	3.740C	1306853W8

$13,260,000 4.125% 2012 Series B Term Bonds due June 1, 2037, Yield: 4.240%, CUSIP†: 1306853X6

Estimated Sources and Uses of Funds

The proceeds to be received from the sale of the 2012B Bonds are expected to be applied as set forth below:

Estimated Sources	
Principal Amount of 2012B Bonds	$42,050,000.00
Plus Net Original Issue Premium	3,789,642.70
Total Estimated Sources	$45,839,642.70
Estimated Uses	
Project Account	$39,595,000.00
Capitalized Interest[(1)]	5,931,575.09
Costs of Issuance[(2)]	131,795.97
Underwriters' Discount	181,271.64
Total Estimated Uses	$45,839,642.70

(1) *Funded to pay interest on the 2012B Bonds to the date which is three months after the expected construction completion date for the 2012B Project.*
(2) *Includes the State Treasurer's fees for serving as trustee, legal and rating agencies' fees, and other costs of issuance, including Board administration fees.*

Annual Fiscal Year Debt Service Requirements

Set forth below are the principal, interest and total debt service requirements for the 2012B Bonds, assuming no redemptions other than scheduled mandatory sinking account redemptions:

Payment Date	2012B Bonds Principal	2012B Bonds Interest	Total 2012B Bonds Debt Service	Annual Fiscal Year Debt Service
12/1/2012		$1,143,775.85	$1,143,775.85	
6/1/2013		957,549.24	957,549.24	$2,101,325.09
12/1/2013		957,575.76	957,575.76	
6/1/2014		957,549.24	957,549.24	1,915,125.00
12/1/2014		957,575.76	957,575.76	
6/1/2015	$1,075,000	957,549.24	2,032,549.24	2,990,125.00
12/1/2015		936,075.76	936,075.76	
6/1/2016	1,120,000	936,049.24	2,056,049.24	2,992,125.00
12/1/2016		913,675.76	913,675.76	
6/1/2017	1,165,000	913,649.24	2,078,649.24	2,992,325.00
12/1/2017		890,375.76	890,375.76	
6/1/2018	1,210,000	890,349.24	2,100,349.24	2,990,725.00
12/1/2018		866,175.76	866,175.76	
6/1/2019	1,255,000	866,149.24	2,121,149.24	2,987,325.00
12/1/2019		841,075.76	841,075.76	
6/1/2020	1,310,000	841,049.24	2,151,049.24	2,992,125.00
12/1/2020		814,875.76	814,875.76	
6/1/2021	1,360,000	814,849.24	2,174,849.24	2,989,725.00
12/1/2021		780,875.76	780,875.76	
6/1/2022	1,430,000	780,849.24	2,210,849.24	2,991,725.00
12/1/2022		745,125.76	745,125.76	
6/1/2023	1,500,000	745,099.24	2,245,099.24	2,990,225.00
12/1/2023		707,625.76	707,625.76	
6/1/2024	1,575,000	707,599.24	2,282,599.24	2,990,225.00
12/1/2024		668,250.76	668,250.76	
6/1/2025	1,655,000	668,224.24	2,323,224.24	2,991,475.00
12/1/2025		626,875.76	626,875.76	
6/1/2026	1,735,000	626,849.24	2,361,849.24	2,988,725.00
12/1/2026		583,500.76	583,500.76	
6/1/2027	1,825,000	583,474.24	2,408,474.24	2,991,975.00
12/1/2027		537,875.76	537,875.76	
6/1/2028	1,915,000	537,849.24	2,452,849.24	2,990,725.00
12/1/2028		490,000.76	490,000.76	
6/1/2029	2,010,000	489,974.24	2,499,974.24	2,989,975.00
12/1/2029		439,750.76	439,750.76	
6/1/2030	2,110,000	439,724.24	2,549,724.24	2,989,475.00
12/1/2030		387,000.76	387,000.76	
6/1/2031	2,215,000	386,974.24	2,601,974.24	2,988,975.00
12/1/2031		331,625.76	331,625.76	
6/1/2032	2,325,000	331,599.24	2,656,599.24	2,988,225.00
12/1/2032		273,500.76	273,500.76	
6/1/2033	2,440,000	273,474.24	2,713,474.24	2,986,975.00
12/1/2033		223,173.32	223,173.32	
6/1/2034	2,545,000	223,151.68	2,768,151.68	2,991,325.00
12/1/2034		170,680.15	170,680.15	
6/1/2035	2,650,000	170,663.60	2,820,663.60	2,991,343.75
12/1/2035		116,021.25	116,021.25	
6/1/2036	2,755,000	116,010.00	2,871,010.00	2,987,031.25
12/1/2036		59,196.62	59,196.62	
6/1/2037	2,870,000	59,190.88	2,929,190.88	2,988,387.50
	$42,050,000	$30,737,712.59	$72,787,712.59	$72,787,712.59

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Start with a Basic Loan…

Date	Principal Balance	Principal	5.00% Interest on Balance	Debt Service
5/1/2014	50,000,000			
5/1/2015	50,000,000	9,048,740	2,500,000	11,548,740
5/1/2016	40,951,260	9,501,177	2,047,563	11,548,740
5/1/2017	31,450,083	9,976,236	1,572,504	11,548,740
5/1/2018	21,473,847	10,475,048	1,073,692	11,548,740
5/1/2019	10,998,800	10,998,800	549,940	11,548,740
Total		50,000,000	7,743,700	57,743,700

Assumptions –

- $50,000,000 borrowed
- Repaid in 5 years
- Interest rate of 5.00%

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…Round by Denominations…

Date	Principal Balance	Principal	5.00% Interest on Balance	Debt Service
5/1/2014	50,000,000			
5/1/2015	50,000,000	9,050,000	2,500,000	11,550,000
5/1/2016	40,950,000	9,500,000	2,047,500	11,547,500
5/1/2017	31,450,000	9,975,000	1,572,500	11,547,500
5/1/2018	21,475,000	10,475,000	1,073,750	11,548,750
5/1/2019	11,000,000	11,000,000	550,000	11,550,000
Total		50,000,000	7,743,750	57,743,750

Municipal bonds are generally sold (and therefore repaid) in denominations of $5,000

…Reflect Different Interest Rates (Coupons) for Each Maturity…

				2.00%	3.00%	4.00%	5.00%	5.00%		
	Principal			Interest on Principal Due					Total	Debt
Date	Balance	Principal	Coupon	5/1/2015	5/1/2016	5/1/2017	5/1/2018	5/1/2019	Interest	Service
5/1/2014	50,000,000									
5/1/2015	50,000,000	9,050,000	2.00%	181,000	285,000	399,000	523,750	550,000	1,938,750	10,988,750
5/1/2016	40,950,000	9,500,000	3.00%		285,000	399,000	523,750	550,000	1,757,750	11,257,750
5/1/2017	31,450,000	9,975,000	4.00%			399,000	523,750	550,000	1,472,750	11,447,750
5/1/2018	21,475,000	10,475,000	5.00%				523,750	550,000	1,073,750	11,548,750
5/1/2019	11,000,000	11,000,000	5.00%					550,000	550,000	11,550,000
Total		50,000,000		181,000	570,000	1,197,000	2,095,000	2,750,000	6,793,000	56,793,000

Or in the more familiar form below:

	Principal			Total	Debt
Date	Balance	Principal	Coupon	Interest	Service
5/1/2014	50,000,000				
5/1/2015	50,000,000	9,050,000	2.00%	1,938,750	10,988,750
5/1/2016	40,950,000	9,500,000	3.00%	1,757,750	11,257,750
5/1/2017	31,450,000	9,975,000	4.00%	1,472,750	11,447,750
5/1/2018	21,475,000	10,475,000	5.00%	1,073,750	11,548,750
5/1/2019	11,000,000	11,000,000	5.00%	550,000	11,550,000
Total		50,000,000		6,793,000	56,793,000

…Adjust Principal of Each Maturity to Achieve Debt Service Pattern…

				2.00%	3.00%	4.00%	5.00%	5.00%		
	Principal			Interest on Principal Due					Total	Debt
Date	Balance	Principal	Coupon	5/1/2015	5/1/2016	5/1/2017	5/1/2018	5/1/2019	Interest	Service
5/1/2014	50,000,000									
5/1/2015	50,000,000	9,415,000	2.00%	188,300	288,150	395,600	514,500	540,000	1,926,550	11,341,550
5/1/2016	40,585,000	9,605,000	3.00%		288,150	395,600	514,500	540,000	1,738,250	11,343,250
5/1/2017	30,980,000	9,890,000	4.00%			395,600	514,500	540,000	1,450,100	11,340,100
5/1/2018	21,090,000	10,290,000	5.00%				514,500	540,000	1,054,500	11,344,500
5/1/2019	10,800,000	10,800,000	5.00%					540,000	540,000	11,340,000
Total		50,000,000		188,300	576,300	1,186,800	2,058,000	2,700,000	6,709,400	56,709,400

Once again, or in the more familiar form below:

	Principal			Total	Debt
Date	Balance	Principal	Coupon	Interest	Service
5/1/2014	50,000,000				
5/1/2015	50,000,000	9,415,000	2.00%	1,926,550	11,341,550
5/1/2016	40,585,000	9,605,000	3.00%	1,738,250	11,343,250
5/1/2017	30,980,000	9,890,000	4.00%	1,450,100	11,340,100
5/1/2018	21,090,000	10,290,000	5.00%	1,054,500	11,344,500
5/1/2019	10,800,000	10,800,000	5.00%	540,000	11,340,000
Total		50,000,000		6,709,400	56,709,400

…Introduce Prices, Yields and Proceeds…

Date	Principal Balance	Principal	Coupon	Total Interest	Debt Service	Yield	Price	Proceeds
5/1/2014	50,000,000							
5/1/2015	50,000,000	9,415,000	2.00%	1,926,550	11,341,550	1.00%	100.992	9,508,397
5/1/2016	40,585,000	9,605,000	3.00%	1,738,250	11,343,250	1.75%	102.446	9,839,938
5/1/2017	30,980,000	9,890,000	4.00%	1,450,100	11,340,100	2.25%	105.049	10,389,346
5/1/2018	21,090,000	10,290,000	5.00%	1,054,500	11,344,500	2.75%	108.467	11,161,254
5/1/2019	10,800,000	10,800,000	5.00%	540,000	11,340,000	3.10%	108.737	11,743,596
Total		50,000,000		6,709,400	56,709,400			52,642,532

Each maturity generates proceeds equal to the product of its price and its principal.

Note: Prices are calculated following all of the rules discussed above.

CALIFORNIA DEBT AND INVESTMENT ADVISORY COMMISSION
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…Calculate Purchase Price…

Date	Principal Balance	Principal	Coupon	Total Interest	Debt Service	Yield	Price	Proceeds	Takedown ($/$1,000)	Takedown ($)
5/1/2014	50,000,000									
5/1/2015	50,000,000	9,415,000	2.00%	1,926,550	11,341,550	1.00%	100.992	9,508,397	1.00	9,415
5/1/2016	40,585,000	9,605,000	3.00%	1,738,250	11,343,250	1.75%	102.446	9,839,938	2.50	24,013
5/1/2017	30,980,000	9,890,000	4.00%	1,450,100	11,340,100	2.25%	105.049	10,389,346	2.50	24,725
5/1/2018	21,090,000	10,290,000	5.00%	1,054,500	11,344,500	2.75%	108.467	11,161,254	3.75	38,588
5/1/2019	10,800,000	10,800,000	5.00%	540,000	11,340,000	3.10%	108.737	11,743,596	3.75	40,500
Total		50,000,000		6,709,400	56,709,400			52,642,532		137,240

Principal	50,000,000
Net Original Issue Premium/(Discount)	2,642,532
Production	52,642,532
Underwriter's Discount	-162,052
Purchase Price	52,480,479

Underwriter's Discount	
Takedown	137,240
Underwriter's Counsel	15,000
CDIAC	3,000
CUSIP	600
Day Loan	1,462
Dalcomp	3,750
Dalnet	500
DTC	500
Total Uses of Funds	162,052

The purchase price paid to the issuer is net of both compensation and expenses withheld by the underwriter

CALIFORNIA DEBT AND INVESTMENT ADVISORY COMMISSION
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…Add in Sources and Uses Components…

Date	Principal Balance	Principal	Coupon	Total Interest	Debt Service	Yield	Price	Proceeds	Takedown ($/$1,000)	Takedown ($)
5/1/2014	50,000,000									
5/1/2015	50,000,000	9,415,000	2.00%	1,926,550	11,341,550	1.00%	100.992	9,508,397	1.00	9,415
5/1/2016	40,585,000	9,605,000	3.00%	1,738,250	11,343,250	1.75%	102.446	9,839,938	2.50	24,013
5/1/2017	30,980,000	9,890,000	4.00%	1,450,100	11,340,100	2.25%	105.049	10,389,346	2.50	24,725
5/1/2018	21,090,000	10,290,000	5.00%	1,054,500	11,344,500	2.75%	108.467	11,161,254	3.75	38,588
5/1/2019	10,800,000	10,800,000	5.00%	540,000	11,340,000	3.10%	108.737	11,743,596	3.75	40,500
Total		50,000,000		6,709,400	56,709,400			52,642,532		137,240

Sources of Funds	
Principal	50,000,000
Net OIP / (OID)	2,642,532
Funds on Hand	1,000,000
Total Sources of Funds	53,642,532

Uses of Funds	
Project Deposit	50,000,000
Reserve Fund	5,264,253
Costs of Issuance	240,000
Underwriter's Discount	162,052
Contingency	-2,023,774
Total Uses of Funds	53,642,532

Principal	50,000,000
Net Original Issue Premium/(Discount)	2,642,532
Production	52,642,532
Underwriter's Discount	-162,052
Purchase Price	52,480,479

Underwriter's Discount	
Takedown	137,240
Underwriter's Counsel	15,000
CDIAC	3,000
CUSIP	600
Day Loan	1,462
Dalcomp	3,750
Dalnet	500
DTC	500
Total Uses of Funds	162,052

Project deposit represents target amount to be borrowed

Notes: Reserve fund is generally equal to the least of: 10% of proceeds, maximum annual debt service (MADS) and 125% of average annual debt service. Contingency is a positive number that is less than the minimum denomination, adjusted by the prices of the bonds

…and Readjust Principal of Each Maturity to Target Proceeds

Date	Principal Balance	Principal	Coupon	Total Interest	Debt Service	Yield	Price	Proceeds	Takedown ($/$1,000)	Takedown ($)
5/1/2014	50,000,000									
5/1/2015	50,000,000	9,820,000	2.00%	2,009,200	11,829,200	1.00%	100.992	9,917,414	1.00	9,820
5/1/2016	40,180,000	10,015,000	3.00%	1,812,800	11,827,800	1.75%	102.446	10,259,967	2.50	25,038
5/1/2017	30,165,000	10,315,000	4.00%	1,512,350	11,827,350	2.25%	105.049	10,835,804	2.50	25,788
5/1/2018	19,850,000	10,730,000	5.00%	1,099,750	11,829,750	2.75%	108.467	11,638,509	3.75	40,238
5/1/2019	9,120,000	11,265,000	5.00%	563,250	11,828,250	3.10%	108.737	12,249,223	3.75	42,244
Total		52,145,000		6,997,350	59,142,350			54,900,918		143,126

Sources of Funds	
Principal	52,145,000
Net OIP / (OID)	2,755,918
Funds on Hand	1,000,000
Total Sources of Funds	55,900,918

Uses of Funds	
Project Deposit	50,000,000
Reserve Fund	5,490,092
Costs of Issuance	240,000
Underwriter's Discount	168,162
Contingency	2,664
Total Uses of Funds	55,900,918

Principal	52,145,000
Net Original Issue Premium/(Discount)	2,755,918
Production	54,900,918
Underwriter's Discount	-168,162
Purchase Price	54,732,756

Underwriter's Discount	
Takedown	143,126
Underwriter's Counsel	15,000
CDIAC	3,000
CUSIP	600
Day Loan	1,525
Dalcomp	3,911
Dalnet	500
DTC	500
Total Uses of Funds	168,162

Note: Contingency should be greater than zero, but less than one denomination of the issued bond, after accounting for the prices of the bonds.

CALIFORNIA DEBT AND INVESTMENT ADVISORY COMMISSION
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How to Calculate the "Yield" of a Bond Issue

Date	Principal Balance	Principal	Coupon	Total Interest	Debt Service	Present Value of Debt Service at 2.55204%	2.65880%	2.81677%
5/1/2014	50,000,000							
5/1/2015	50,000,000	9,820,000	2.00%	2,009,200	11,829,200	11,545,493	11,533,854	11,516,666
5/1/2016	40,180,000	10,015,000	3.00%	1,812,800	11,827,800	11,253,871	11,230,634	11,196,363
5/1/2017	30,165,000	10,315,000	4.00%	1,512,350	11,827,350	10,969,879	10,935,624	10,885,168
5/1/2018	19,850,000	10,730,000	5.00%	1,099,750	11,829,750	10,694,982	10,650,256	10,584,464
5/1/2019	9,120,000	11,265,000	5.00%	563,250	11,828,250	10,422,842	10,368,195	10,287,917
Total		52,145,000		6,997,350	59,142,350	54,887,069	54,718,564	54,470,579

		Arbitrage Yield	True Interest Cost	All-in True Interest Cost
Sources of Funds				
Principal	52,145,000			
Net OIP / (OID)	2,755,918			
Funds on Hand	1,000,000			
Total Sources of Funds	55,900,918			
Uses of Funds				
Project Deposit	50,000,000			
Reserve Fund	5,490,092			
Costs of Issuance	240,000			
Underwriter's Discount	168,162			
Contingency	2,664			
Total Uses of Funds	55,900,918			
		Proceeds	54,900,918	54,900,918	54,900,918
		Costs of Issuance			-240,000
		Underwriter's Discount		-168,162	-168,162
		Arbitrage Adjustments			
		Target Value	54,900,918	54,732,756	54,492,756

Find the rate as the internal rate of return (IRR) of debt service to the target value

Note: Debt service may be required to be adjusted for bonds subject to redemption, when calculating the arbitrage yield

Arbitrage yield, true interest cost (TIC) and all-in TIC each represent a way to express the cost of capital for a bond issue

How to Calculate an "Average"

Date	Principal Balance	Principal	Coupon	Princpial x Coupon	Total Interest	Debt Service	Years to Maturity	Principal x Years to Maturity
5/1/2014	50,000,000							
5/1/2015	50,000,000	9,820,000	2.00%	196,400	2,009,200	11,829,200	1.00000	9,820,000
5/1/2016	40,180,000	10,015,000	3.00%	300,450	1,812,800	11,827,800	2.00000	20,030,000
5/1/2017	30,165,000	10,315,000	4.00%	412,600	1,512,350	11,827,350	3.00000	30,945,000
5/1/2018	19,850,000	10,730,000	5.00%	536,500	1,099,750	11,829,750	4.00000	42,920,000
5/1/2019	9,120,000	11,265,000	5.00%	563,250	563,250	11,828,250	5.00000	56,325,000
Total		52,145,000		2,009,200	6,997,350	59,142,350		160,040,000

Average Coupon
2,009,200
52,145,000
3.85%

Weighted Average Maturity
160,040,000
52,145,000
3.06913

In general, averages are calculated as weighted averages by principal

Debt Amortization

Bond cash flows literacy

intermediate bond math (Part 1)

CALIFORNIA DEBT AND INVESTMENT ADVISORY COMMISSION
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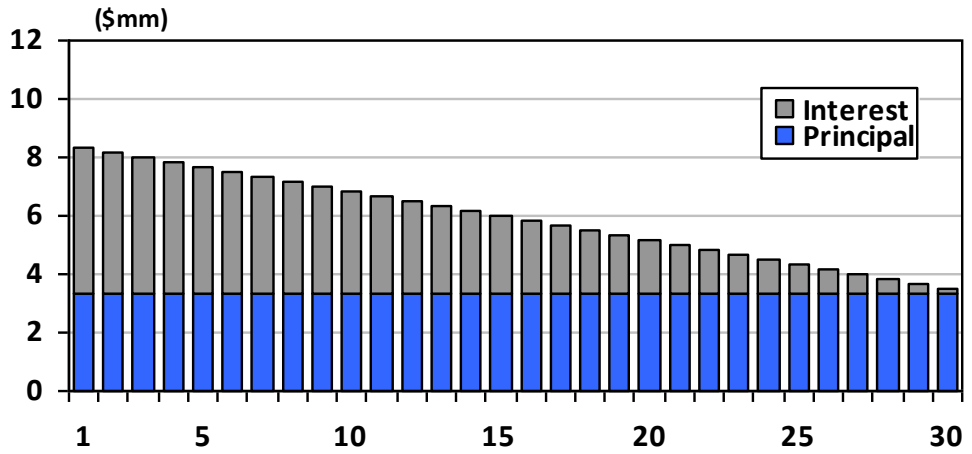
Common Amortization Structures

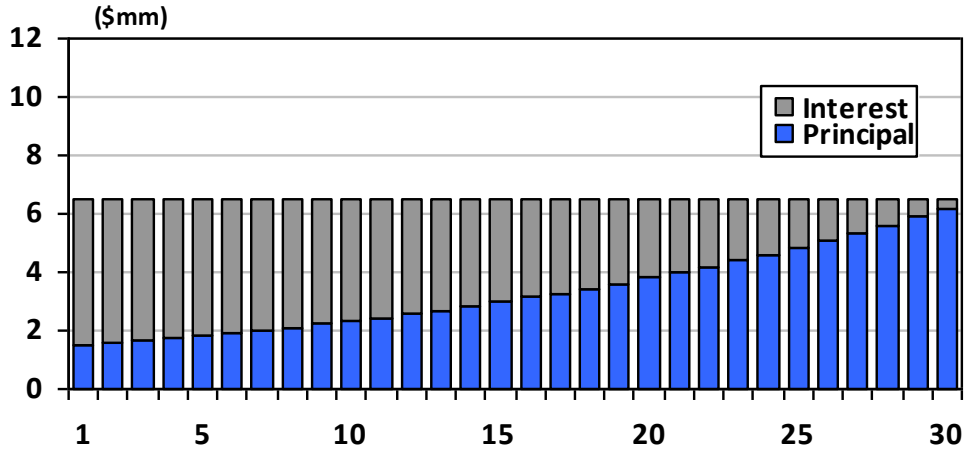
Level Principal:

- *Ease of calculation*
- *Common for bank product term-out provisions and GOs*
- *Interest/principal ratio: 0.77*
  (based on 5% rate)



Level Debt Service:

- *Even distribution of cost*
- *Simplify long-term budget preparation*
- *Interest/principal ratio: 0.95*
  (based on 5% rate)



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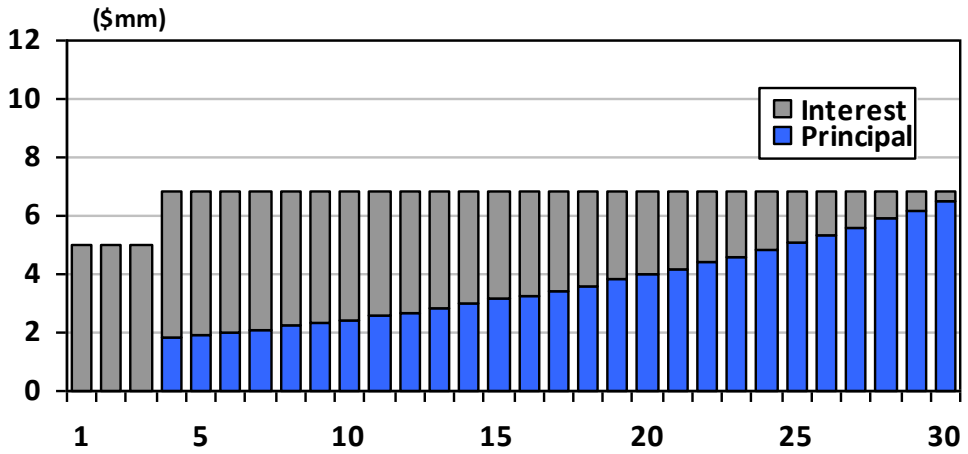
Common Amortization Structures

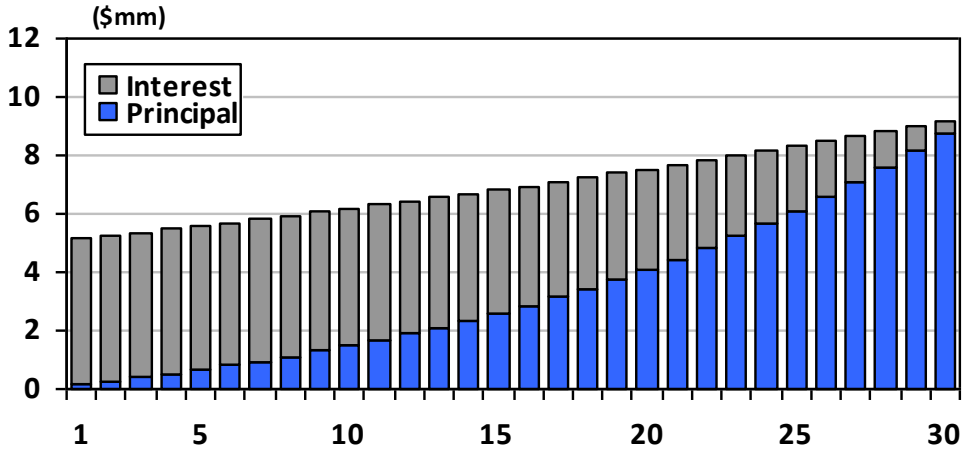
Deferred Principal:

- *Revenues or operational cost savings become available at the later date (e.g. upon project completion)*
- *Interest/principal ratio: 0.99*
  (based on 5% rate)



Ascending Debt Service:

- *Growing revenues*
- *Cost-recovery mechanism is subject to inflation*
- *Interest/principal ratio: 1.10*
  (based on 5% rate and 2% annual growth)



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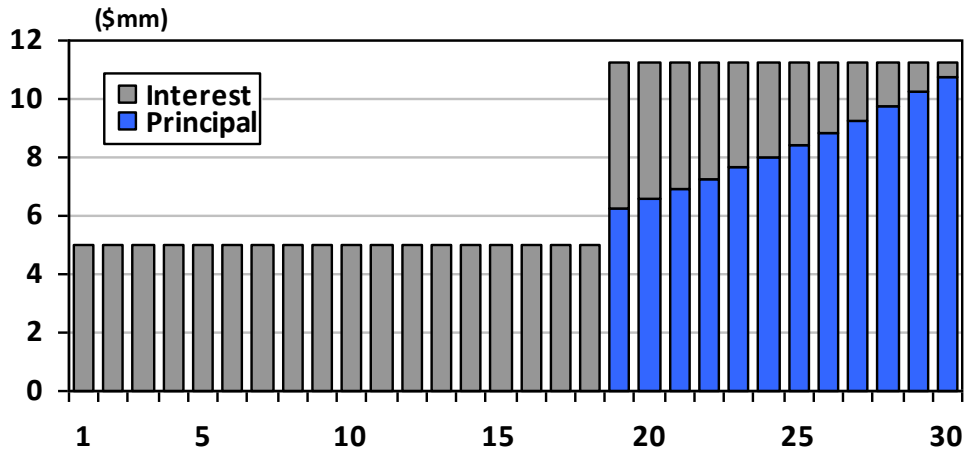
Common Amortization Structures

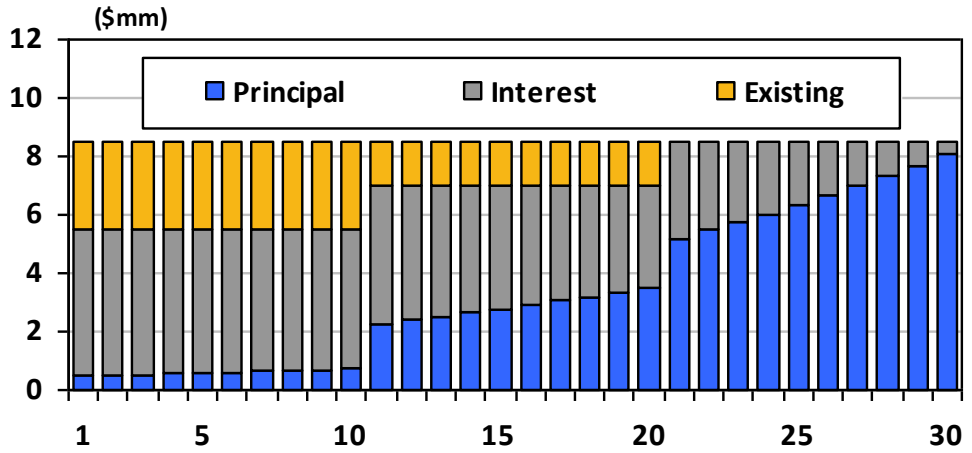
Backloaded Principal:

- *Type of bond has the lowest expected cost of funds (e.g., floating rate or tax credit bonds)*
- *Interest/principal ratio: 1.25*
  (based on 5% rate, for THIS example)



Wrapped Debt Service:

- *Profile of aggregate debt service is level*
- *Extends useful life of existing debt-funded asset*
- *Interest/principal ratio: 1.09*
  (based on 5% rate, for THIS example)



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Solving for Amortization Structure

Debt service is equal to the sum of:

- *Principal;*
- *Interest on principal due; and*
- *Interest on principal still outstanding*

- Debt Service for a given year:

$$DS_n = (1 + C_n) \times P_n + \sum_{i=n+1}^{`} P_i \times C_i$$

  - "DS_n" = Debt service for year n
  - "P_n" = Principal amount for maturity n
  - "C_n" = Coupon for maturity n

- Solving algebraically for principal results in the following:

$$P_n = \frac{DS_n - \sum_{i=n+1}^{`} P_i \times C_i}{1 + C_n}$$

- Given target debt service numbers, each principal amount can be solved

CALIFORNIA DEBT AND INVESTMENT ADVISORY COMMISSION
presented by: PRAG

Solving for Amortization Structure

No unknowns!

- Begin from the last maturity…

Example:

Target debt service: $5,000,000
Coupon for 2024 (last maturity): 5.00%

$$P_{2018} = \frac{\$5{,}000{,}000 - \$0}{1 + 5.00\%}$$

$$P_{2018} = \$4{,}761{,}904.80$$ or round down to

$$P_{2018} = \$4{,}760{,}000.00$$

Rounding based on denomination

Year	Target Debt Service	Principal	Coupon	Interest	Debt Service	Difference between Target and Actual D/S
2015	$5,000,000	????	3.00%	238,000	238,000	4,762,000
2016	$5,000,000	????	3.00%	238,000	238,000	4,762,000
2017	$5,000,000	????	3.50%	238,000	238,000	4,762,000
2018	$5,000,000	????	5.00%	238,000	238,000	4,762,000
2019	$5,000,000	????	5.00%	238,000	238,000	4,762,000
2020	$5,000,000	????	5.00%	238,000	238,000	4,762,000
2021	$5,000,000	????	5.00%	238,000	238,000	4,762,000
2022	$5,000,000	????	4.75%	238,000	238,000	4,762,000
2023	$5,000,000	????	4.75%	238,000	238,000	4,762,000
2024	$5,000,000	4,760,000	5.00%	238,000	4,998,000	2,000
Total	$50,000,000	4,760,000		2,380,000	7,140,000	42,860,000

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Solving for Amortization Structure (Cont'd)

…only one unknown…

- …Continue with next to last maturity…

Example:

Target debt service : $5,000,000
Coupon for 2023 (next to last maturity): 4.75%
Principal for 2024 (last maturity): $4,760,000
Coupon for 2024 (last maturity): 5.00%

$$P_{2017} = \frac{\$5{,}000{,}000 - \$4{,}760{,}000 \times 5.00\%}{1 + 4.75\%}$$

$$P_{2017} = \$4{,}546{,}062.05$$ or round down to

$$P_{2017} = \$4{,}545{,}000.00$$

…which was just solved in the last step!

Year	Target Debt Service	Principal	Coupon	Interest	Debt Service	Difference between Target and Actual D/S
2015	$5,000,000	????	3.00%	$453,888	$453,888	$4,546,113
2016	$5,000,000	????	3.00%	$453,888	$453,888	$4,546,113
2017	$5,000,000	????	3.50%	$453,888	$453,888	$4,546,113
2018	$5,000,000	????	5.00%	$453,888	$453,888	$4,546,113
2019	$5,000,000	????	5.00%	$453,888	$453,888	$4,546,113
2020	$5,000,000	????	5.00%	$453,888	$453,888	$4,546,113
2021	$5,000,000	????	5.00%	$453,888	$453,888	$4,546,113
2022	$5,000,000	????	4.75%	$453,888	$453,888	$4,546,113
2023	$5,000,000	$4,545,000	4.75%	$453,888	$4,998,888	$1,113
2024	$5,000,000	$4,760,000	5.00%	$238,000	$4,998,000	$2,000
Total	$50,000,000	$9,305,000		$4,322,988	$13,627,988	$36,372,013

CALIFORNIA DEBT AND INVESTMENT ADVISORY COMMISSION
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Solving for Amortization Structure (Cont'd)

Remaining unknowns will be solved just in time as well

- …And so forth

Year	Target Debt Service	Principal	Coupon	Interest	Debt Service	Difference between Target and Actual D/S
2015	$5,000,000	$3,250,000	3.00%	$1,747,400	$4,997,400	$2,600
2016	$5,000,000	$3,345,000	3.00%	$1,649,900	$4,994,900	$5,100
2017	$5,000,000	$3,450,000	3.50%	$1,549,550	$4,999,550	$450
2018	$5,000,000	$3,570,000	5.00%	$1,428,800	$4,998,800	$1,200
2019	$5,000,000	$3,745,000	5.00%	$1,250,300	$4,995,300	$4,700
2020	$5,000,000	$3,935,000	5.00%	$1,063,050	$4,998,050	$1,950
2021	$5,000,000	$4,130,000	5.00%	$866,300	$4,996,300	$3,700
2022	$5,000,000	$4,335,000	4.75%	$659,800	$4,994,800	$5,200
2023	$5,000,000	$4,545,000	4.75%	$453,888	$4,998,888	$1,113
2024	$5,000,000	$4,760,000	5.00%	$238,000	$4,998,000	$2,000
Total	$50,000,000	$39,065,000		$10,906,988	$49,971,988	$28,013

Adjusting for Target Proceeds

If there is too much principal (or if there are too many proceeds), reduce target debt service

If there is too little principal (or if there are too few proceeds), increase target debt service

- To solve for a target par or proceeds amount:
  - Make an initial guess for target debt service
  - Rescale accordingly

$$DS_{T,1} = \frac{DS_{T,0} \times P_T}{P_0}$$

    - ✓ "$DS_{T,1}$" = New target debt service
    - ✓ "$DST_{T,0}$" = Initial target debt service
    - ✓ "P_T" = Target par amount
    - ✓ "P_0" = Par amount from initial target debt service
  - Iterate, if necessary

$$DS_{T,n} = \frac{DS_{T,n-1} \times P_T}{P_{n-1}}$$

  - It may be necessary to adjust by taking the average when within one denomination

$$DS_{T,n} = \left(\frac{DS_{T,n-1}}{P_{n-1}} + \frac{DS_{T,n-2}}{P_{n-2}} \right) \times \frac{P_T}{2}$$

Adjusting for Target Proceeds (Cont'd)

Example:

Target principal: $40,000,000

Coupons: As shown below

Initial target debt service: $5,000,000

Year	Target Debt Service	Principal	Coupon	Interest	Debt Service	Difference between Target and Actual D/S
2015	$5,000,000	$3,250,000	3.00%	$1,747,400	$4,997,400	$2,600
2016	$5,000,000	$3,345,000	3.00%	$1,649,900	$4,994,900	$5,100
2017	$5,000,000	$3,450,000	3.50%	$1,549,550	$4,999,550	$450
2018	$5,000,000	$3,570,000	5.00%	$1,428,800	$4,998,800	$1,200
2019	$5,000,000	$3,745,000	5.00%	$1,250,300	$4,995,300	$4,700
2020	$5,000,000	$3,935,000	5.00%	$1,063,050	$4,998,050	$1,950
2021	$5,000,000	$4,130,000	5.00%	$866,300	$4,996,300	$3,700
2022	$5,000,000	$4,335,000	4.75%	$659,800	$4,994,800	$5,200
2023	$5,000,000	$4,545,000	4.75%	$453,888	$4,998,888	$1,113
2024	$5,000,000	$4,760,000	5.00%	$238,000	$4,998,000	$2,000
Total	$50,000,000	$39,065,000		$10,906,988	$49,971,988	$28,013

$$DS_{T,1} = \frac{\$5,000,000 \times \$40,000,000}{\$39,065,000}$$

$$DS_{T,1} = \$5,119,672.34$$

Adjusting for Target Proceeds (Cont'd)

Example (cont'd):

Target principal: $40,000,000

Coupons: As shown below

Initial target debt service: $5,000,000

Second target debt service: $5,119,672.34

Year	Target Debt Service	Principal	Coupon	Interest	Debt Service	Difference between Target and Actual D/S
2015	$5,119,672	$3,330,000	3.00%	$1,789,375	$5,119,375	$297
2016	$5,119,672	$3,430,000	3.00%	$1,689,475	$5,119,475	$197
2017	$5,119,672	$3,530,000	3.50%	$1,586,575	$5,116,575	$3,097
2018	$5,119,672	$3,655,000	5.00%	$1,463,025	$5,118,025	$1,647
2019	$5,119,672	$3,835,000	5.00%	$1,280,275	$5,115,275	$4,397
2020	$5,119,672	$4,030,000	5.00%	$1,088,525	$5,118,525	$1,147
2021	$5,119,672	$4,230,000	5.00%	$887,025	$5,117,025	$2,647
2022	$5,119,672	$4,440,000	4.75%	$675,525	$5,115,525	$4,147
2023	$5,119,672	$4,650,000	4.75%	$464,625	$5,114,625	$5,047
2024	$5,119,672	$4,875,000	5.00%	$243,750	$5,118,750	$922
Total	$51,196,723	$40,005,000		$11,168,175	$51,173,175	$23,548

Attempt	Target Debt Service	Resultant Principal	Solution Method
1	$5,000,000.00	$39,065,000.00	Rescale
2	5,119,672.34	40,005,000.00	Rescale
3	5,119,032.46	39,995,000.00	Rescale
4	5,119,352.40	40,000,000.00	Average

Bonus: Excel Functions

Bond cash flows literacy

intermediate bond math (Part 1)

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Using PRICE()

- Needs to be supplemented for:
  - Par bonds;
  - Rounding; and
  - Call provisions for premium bonds
- Effective form for bonds callable at par is as follows:

	A	B
1	Delivery	5/14/2014
2	Maturity	5/1/2028
3	Coupon	5.00%
4	Yield	3.65%
5	Call Date1	5/1/2024
6	Call Price1	100

```
=IF(B3=B4,100,TRUNC(PRICE(B1,IF(AND(B3>B4,B2>B5),B5,B2),B3,B4,100,2),3))
```

Check for par bond

Round down to 3 decimal places

Check for premium coupon, using call date as maturity if needed

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Using PRICE()

- For bonds with multiple call prices, must evaluate result for each case

```
=IF(B3=B4,100,TRUNC(MIN(

            PRICE(B1, B2,B3,B4,100,2),    PRICE(B1,
B5,B3,B4,B6,2),
            PRICE(B1, B7,B3,B4,B8,2),
            PRICE(B1, B9,B3,B4,B10,2),

                                                ),3))
```

	A	B
1	Delivery	5/14/2014
2	Maturity	5/1/2028
3	Coupon	5.00%
4	Yield	3.65%
5	Call Date1	5/1/2024
6	Call Price1	102
7	Call Date2	5/1/2025
8	Call Price2	101
9	Call Date3	5/1/2026
10	Call Price3	100

Using EDATE() and EOMONTH()

- Used to create regularly aligned dates for principal amortization or debt service schedules

=EOMONTH(A2,5)+1

=EDATE(A6,6)

	A	B	C	D
1	Date	Principal	Coupon	Interest
2	5/14/2014			
3	11/1/2014			$44,394.17
4	5/1/2015	$1,000,000	2.00%	47,850.00
5	11/1/2015			37,850.00
6	5/1/2016	1,050,000	3.00%	37,850.00
7	11/1/2016			22,100.00
8	5/1/2017	1,105,000	4.00%	22,100.00

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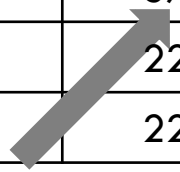
Using SUMPRODUCT()

- Used to calculate interest for entire bond series (with multiple coupons and principal amounts)
  - Tip: Values in last cells must not be blank

	A	B	C	D
1	Date	Principal	Coupon	Interest
2	5/14/2014			
3	11/1/2014			$44,394.17
4	5/1/2015	$1,000,000	2.00%	47,850.00
5	11/1/2015			37,850.00
6	5/1/2016	1,050,000	3.00%	37,850.00
7	11/1/2016			22,100.00
8	5/1/2017	1,105,000	4.00%	22,100.00



=SUMPRODUCT(B6:B$8,C6:C$8)/2

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Using YEARFRAC()

- Used to calculate interest for irregular periods and for ACT/ACT day count basis
  - Tip: Allows for the same formula to be used the cash flow schedule

	A	B	C	D
1	Date	Principal	Coupon	Interest
2	5/14/2014			
3	11/1/2014			$44,394.17
4	5/1/2015	$1,000,000	2.00%	47,850.00
5	11/1/2015			37,850.00
6	5/1/2016	1,050,000	3.00%	37,850.00
7	11/1/2016			22,100.00
8	5/1/2017	1,105,000	4.00%	22,100.00

=SUMPRODUCT(B3:B$8,C3:C$8)*YEARFRAC(A2,A3)

Questions?

Thank you for your participation!

A Certificate of Attendance will be emailed to you within a week.

For MCLE credit, please email cdiac_education@treasurer.ca.gov

The video and transcript of this webinar will be available on CDIAC's website in the near future. Please, contact CDIAC if you would like to be notified when they are posted.